

# General Criteria and Operation Limits of a Steady-State Fusion Reactor with Respect to Plasma-Material Interaction

D. Naujoks

ignition and burn criteria

particle handling

power handling

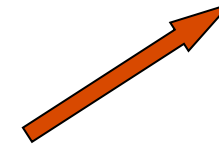
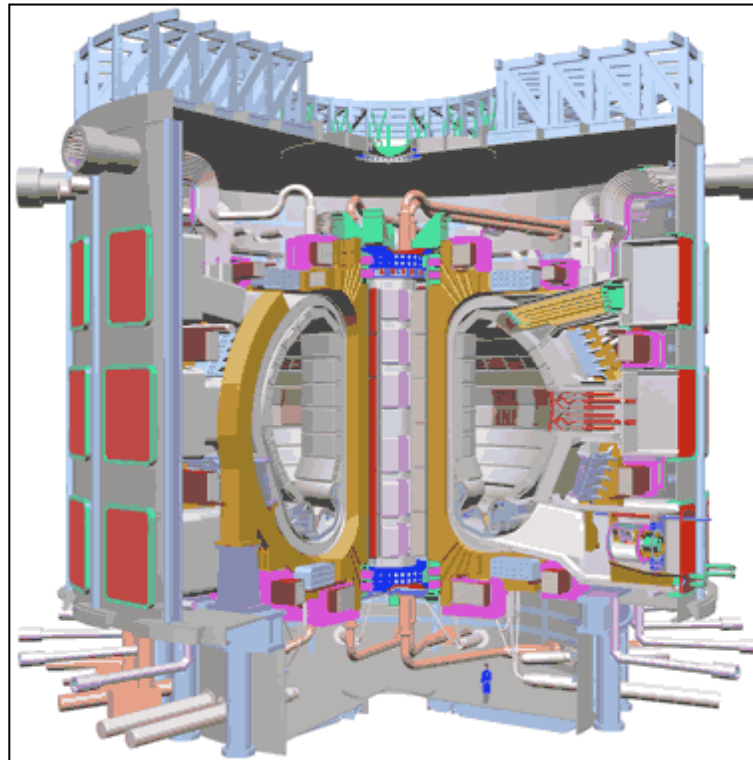
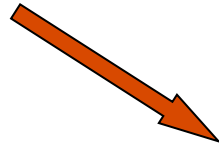
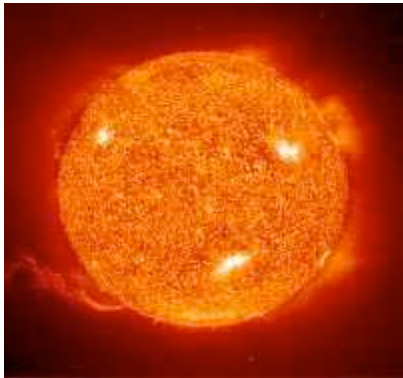
critical impurity concentration

lifetime of wall and divertor elements

choice of material

outlook

# Main Problems of Controlled Fusion

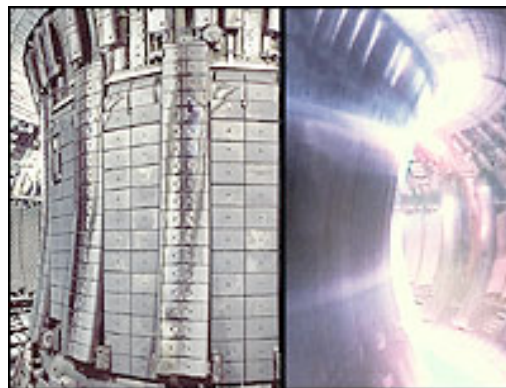


plasma  
confinement

heating  
(ignition and  
burning)

first wall

(plasma operation,  
life time and  
fuel (DT) cycle)



JET

## ignition and burn criteria

particle handling

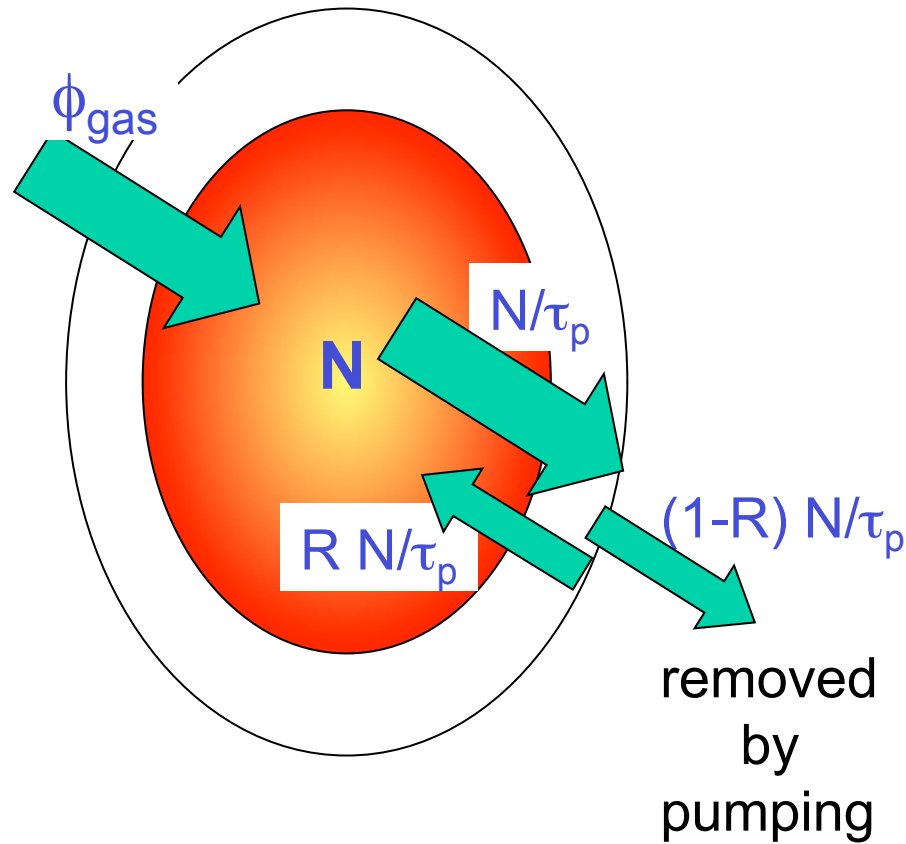
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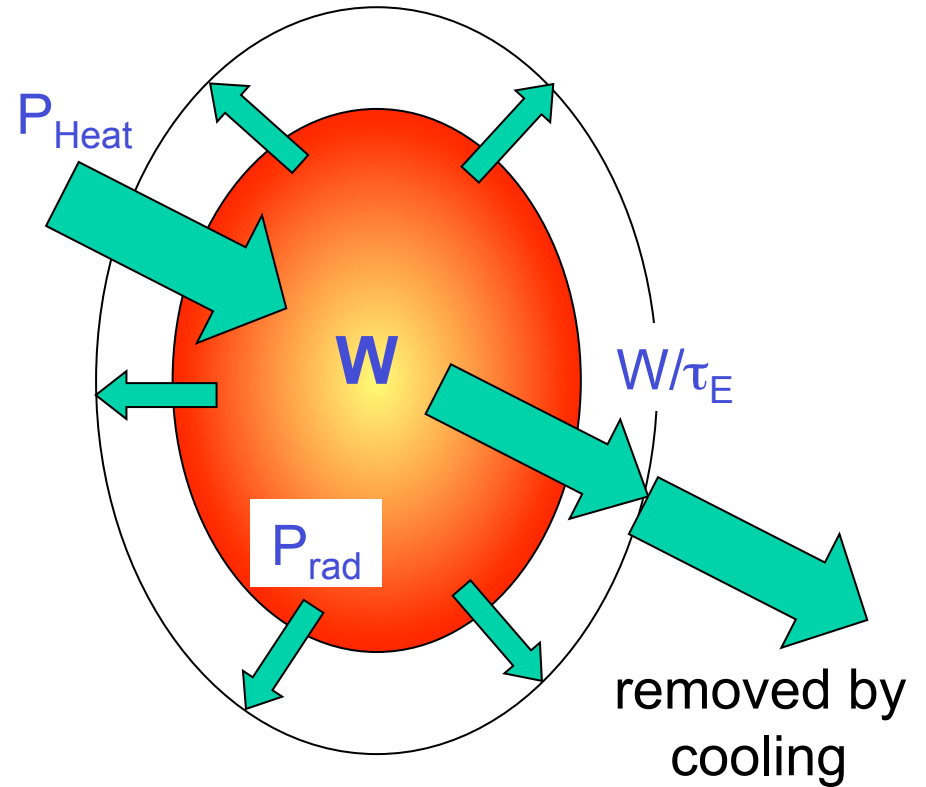
particle balance:

$$\phi_{\text{gas}} = (1 - R) N / \tau_p$$

$\tau_p$

number of plasma

particles:  $N = n V_p$



energy balance:

$$P_{\text{Heat}} = P_{\text{rad}} + W / \tau_E$$

plasma energy:

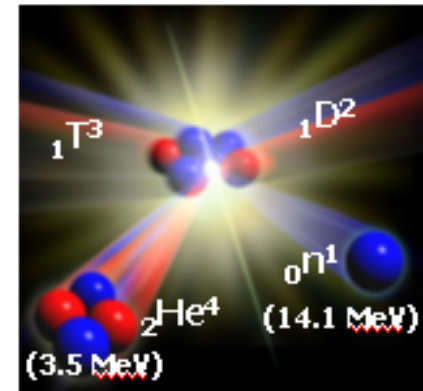
$$W = 3/2 n k (T_e + T_i) V_p$$

ohmic heating is not sufficient  
(only up to 2.5 keV)

$$f_{\text{He}} = n_{\text{He}}/n_e \text{ and } f_i = n_i/n_e$$

charge neutrality and equal densities of deuterium and tritium

$$\sum_i q_i n_i = n_{\text{D}} + n_{\text{T}} + 2n_{\text{He}} + \sum_i q_i n_i = n_e$$



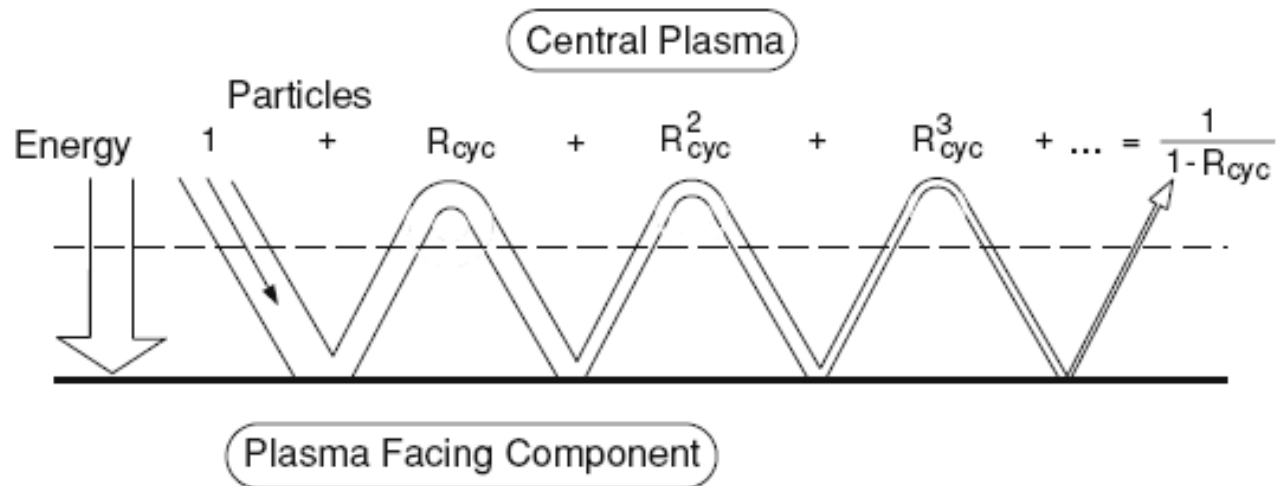
$$n_{\text{DT}} = \frac{n_e}{2} \left( 1 - 2f_{\text{He}} - \sum_i q_i f_i \right) \quad \text{dilution of the DT fusion fuel by helium and impurity ions is characterized by this additional factor}$$

under steady state conditions, the fusion rate (equalizing the production rate of  $\alpha$ -particles) must balance the exhaust rate of helium

$$n_{\text{D}} n_{\text{T}} \langle \sigma v \rangle_{\text{DT}} = \frac{n_{\text{He}}}{\tau_{\text{He}}^*} \quad \tau_{\text{He}} = \frac{\int_{\text{volume}} n_{\text{He}} dV}{\int_{\text{surface}} \Gamma_{\text{He}} dS}$$

the He exhaust itself can be parameterized by introducing the effective confinement time

$$\tau_{\text{He}}^* = \frac{\tau_{\text{He}}}{1 - R_{\text{cyc}}} = \frac{\tau_{\text{He}}}{\epsilon} \quad \epsilon = 1 - R_{\text{cyc}} \text{ is the exhaust efficiency determined by the scrape-off layer (SOL) physics and pumping capabilities}$$



$$\tau_{\text{He}}^* = \frac{\tau_{\text{He}}}{1 - R_{\text{cyc}}} = \frac{\tau_{\text{He}}}{\epsilon}$$

The recycling process: The energy is almost fully absorbed by the material, whereas particles such as He can leave the central plasma region and return many times.

combining the equations

gives the so-called **exhaust criterion**

$$n_{\text{DT}} = \frac{n_e}{2} \left( 1 - 2f_{\text{He}} - \sum q_i f_i \right)$$

$$n_{\text{D}} n_{\text{T}} \langle \sigma v \rangle_{\text{DT}} = \frac{n_{\text{He}}}{\tau_{\text{He}}^*}$$

$$n_e \tau_{\text{He}}^* = \frac{4f_{\text{He}}}{(1 - 2f_{\text{He}} - \sum q_i f_i)^2 \langle \sigma v \rangle_{\text{DT}}}$$

the part of helium fusion power  $P_\alpha$  ( $E_\alpha = 3.52$  MeV) which is not lost through radiation must be confined in the central plasma long enough to replace convective and diffusive heat losses in order to maintain the plasma temperature

$$P_\alpha - P_{\text{brems}} - P_{\text{rad}} = \frac{W_E}{\tau_E} = \frac{3}{2} \left( n_e k_B T_e + 2n_{\text{DT}} k_B T_{\text{DT}} + n_{\text{He}} k_B T_{\text{He}} + \sum n_i k_B T_i \right) / \tau_E \quad \text{energy balance}$$

$$P_\alpha = n_{\text{DT}}^2 \langle \sigma v \rangle_{\text{DT}} E_\alpha = \frac{n_e^2}{4} \left( 1 - 2f_{\text{He}} - \sum f_i q_i \right)^2 \langle \sigma v \rangle_{\text{DT}} E_\alpha \quad \text{helium fusion power}$$

$$P_{\text{rad}} = n_e \sum n_i L_i(T_e) \quad \text{radiation losses}$$

$$P_{\text{brems}} = c_{\text{br}} n_e \left( 2n_{\text{DT}} + 4n_{\text{He}} + \sum_i q_i^2 n_i \right) \sqrt{k_B T_e} = c_{\text{br}} n_e^2 Z_{\text{eff}} \sqrt{k_B T_e}$$

$$= c_{\text{br}} n_e^2 \left( 1 + 2f_{\text{He}} + \sum [q_i^2 - q_i] f_i \right) \sqrt{k_B T_e} \quad \text{bremsstrahlung}$$

$$\left( \text{with } q_{\text{DT}} = 1 \text{ and } q_{\text{He}} = 2 \right) \quad c_{\text{br}} = \frac{16\sqrt{2\pi}\gamma_G}{3\sqrt{3}(4\pi\epsilon_0)^3 m_e^{3/2} c^3 \hbar} = 3.84 \times 10^{-29} \text{ Wm}^2\text{s}/\sqrt{\text{kg}}$$

we obtain the **burn criterion**

$$n_e \tau_E = (3/2) k_B T \left( 2 - f_{\text{He}} + \sum f_i [1 - q_i] \right) / \left[ \left( 1 - 2f_{\text{He}} - \sum f_i q_i \right)^2 \langle \sigma v \rangle_{\text{DT}} E_\alpha / 4 - c_{\text{br}} \left( 1 + 2f_{\text{He}} + \sum [q_i^2 - q_i] f_i \right) \sqrt{k_B T} - \sum f_i L_i \right]$$

the **exhaust criterion** and the **burn criterion** can be combined in order to eliminate the electron density by defining the ratio

$$\gamma = \frac{\tau_{\text{He}}^*}{\tau_E}$$

which results in a **cubic equation** for a certain impurity concentration  $f_i$  for given  $\gamma$ ,  $T$ , and  $f_{\text{He}}$

$$\gamma = \frac{8f_{\text{He}}}{3} \left[ \left( 1 - 2f_{\text{He}} - \sum f_i q_i \right)^2 \langle \sigma v \rangle_{\text{DT}} E_\alpha / 4 - c_{\text{br}} \left( 1 + 2f_{\text{He}} + \sum [q_i^2 - q_i] f_i \right) \sqrt{k_B T} - \sum f_i L_i \right] / \left[ \left( 1 - 2f_{\text{He}} - \sum q_i f_i \right)^2 \times \left( 2 - f_{\text{He}} + \sum f_i [1 - q_i] \right) \langle \sigma v \rangle_{\text{DT}} k_B T \right].$$



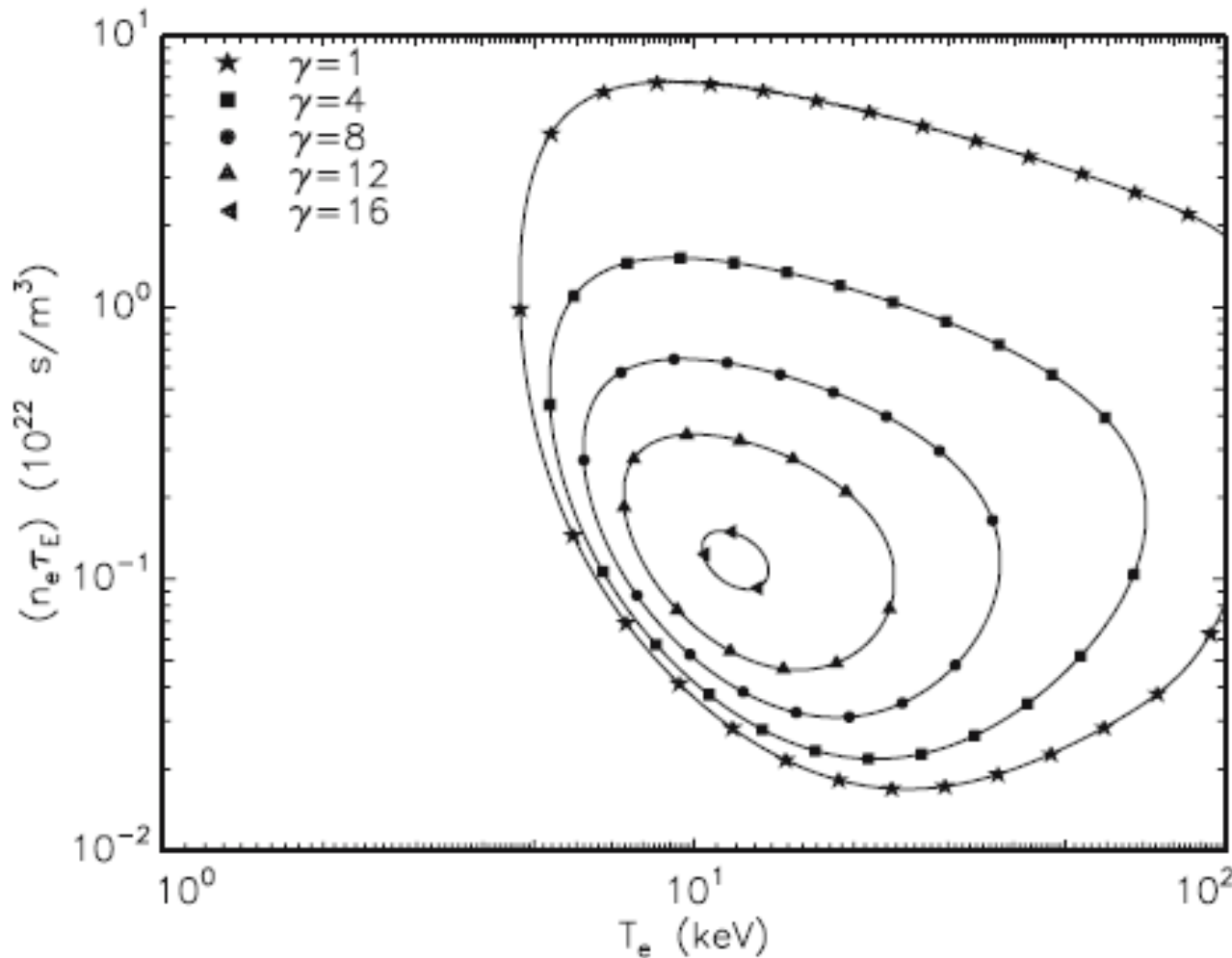
In the case of  $f_i = 0$  (only helium, no other impurities), this reduces to

$$\gamma = \frac{2f_{\text{He}} \langle \sigma v \rangle_{\text{DT}} E_\alpha - 4c_{\text{br}} \sqrt{k_{\text{B}} T} (1 + 2f_{\text{He}}) / (1 - 2f_{\text{He}})^2}{(2 - f_{\text{He}}) \langle \sigma v \rangle_{\text{DT}} k_{\text{B}} T}$$

Usually, this equation has two real solutions,  $f_{\text{He},1}$  and  $f_{\text{He},2}$ , for a given  $\gamma$  value and a certain temperature in the available range of  $0 \leq f_{\text{He}} \leq 1/2$ . Varying the temperature  $T$ , the obtained solutions of can be substituted into the [burn criterion](#). The resulting values of  $n_e \tau_E$  build the workspace of a fusion reactor as a function of  $T$  for a certain value of  $\gamma$ .

solving 
$$\gamma = \frac{2f_{\text{He}} \langle \sigma v \rangle_{\text{DT}} E_{\alpha} - 4c_{\text{br}} \sqrt{k_{\text{B}} T} (1 + 2f_{\text{He}}) / (1 - 2f_{\text{He}})^2}{(2 - f_{\text{He}}) \langle \sigma v \rangle_{\text{DT}} k_{\text{B}} T}$$

$$\gamma = \frac{\tau_{\text{He}}^*}{\tau_{\text{E}}}$$



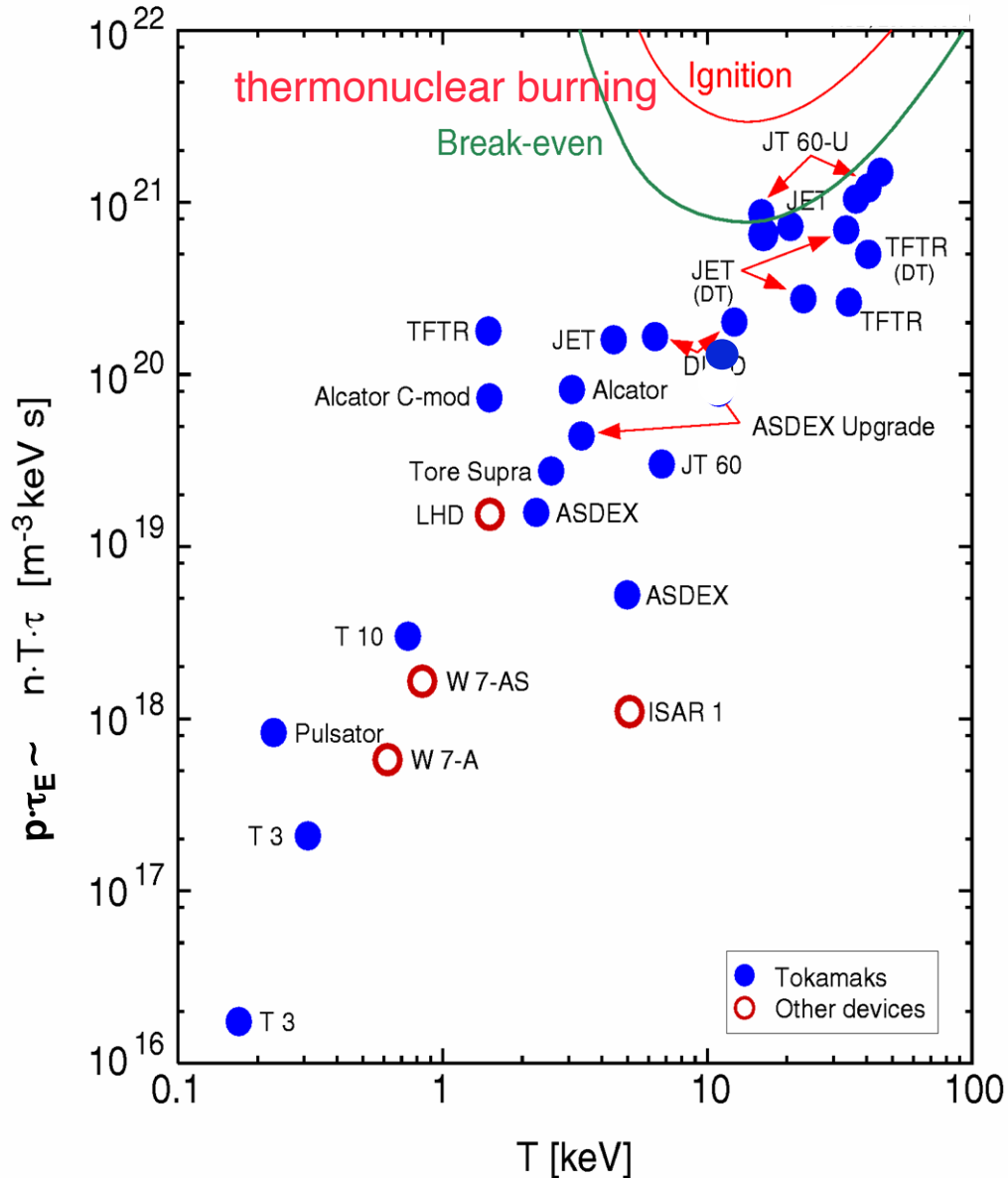
or  $\gamma \geq 16$ , there is no solution at all.

assuming linked transport mechanisms for particle and energy transport, i.e.,  $\tau_{\text{He}} = \tau_{\text{E}}$ , the exhaust efficiency should be larger than  $> 1/16 = 0.06$

$$\tau_{\text{He}}^* = \frac{\tau_{\text{He}}}{1 - R_{\text{cyc}}} = \frac{\tau_{\text{He}}}{\epsilon}$$

this is the helium „ash“ problem

operation space of a fusion reactor considering only helium as an impurity



magnetic confinement

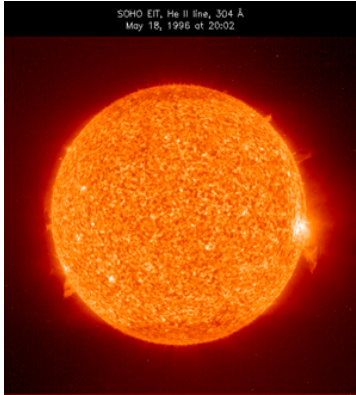
density  $\approx 10^{20} \text{ m}^{-3}$

pressure  $\approx 2\text{-}3 \text{ bar}$

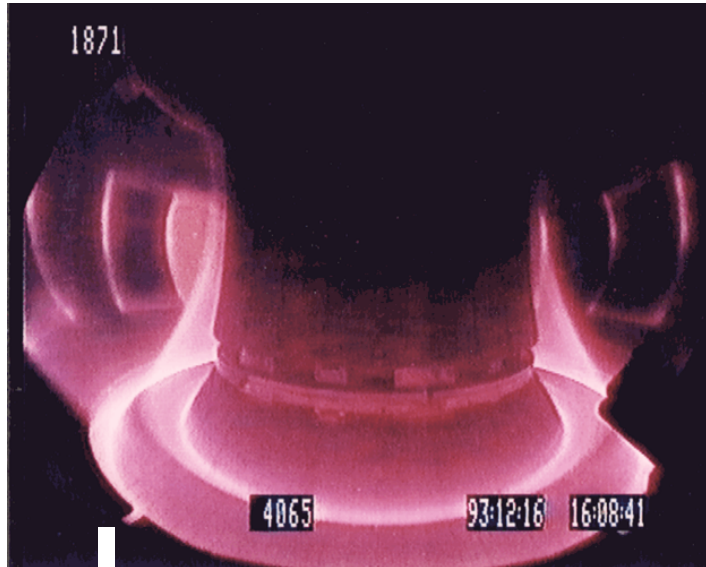
low energy density  $\Leftrightarrow$  secure operation  
but large volume!

	Tokamaks	Helical systems
$T_i(0)$ [keV]	44	4
$T_e(0)$ [keV]	15	7
$n_e(0)$ [ $10^{20} \text{ m}^{-3}$ ]	10	7
$\tau_E$ [s]	1	0.15
$\tau_{\text{plasma}}$ [s]	120	90
$\langle \beta \rangle$ [%]	12.3	2.9
$P_{\text{fusion}}$ [MW]	17	-

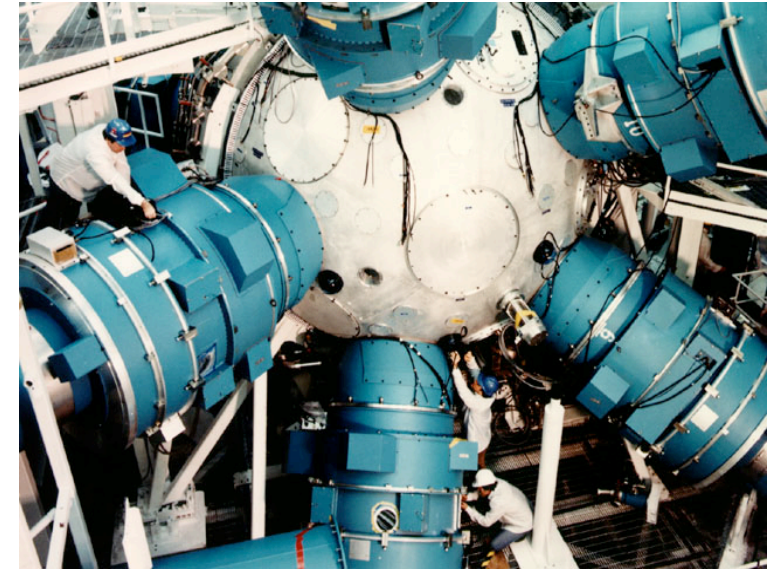
*Table 1 Maximal plasma parameters achieved in tokamaks and stellarators in different discharges.*



Confinement by gravitation: stars



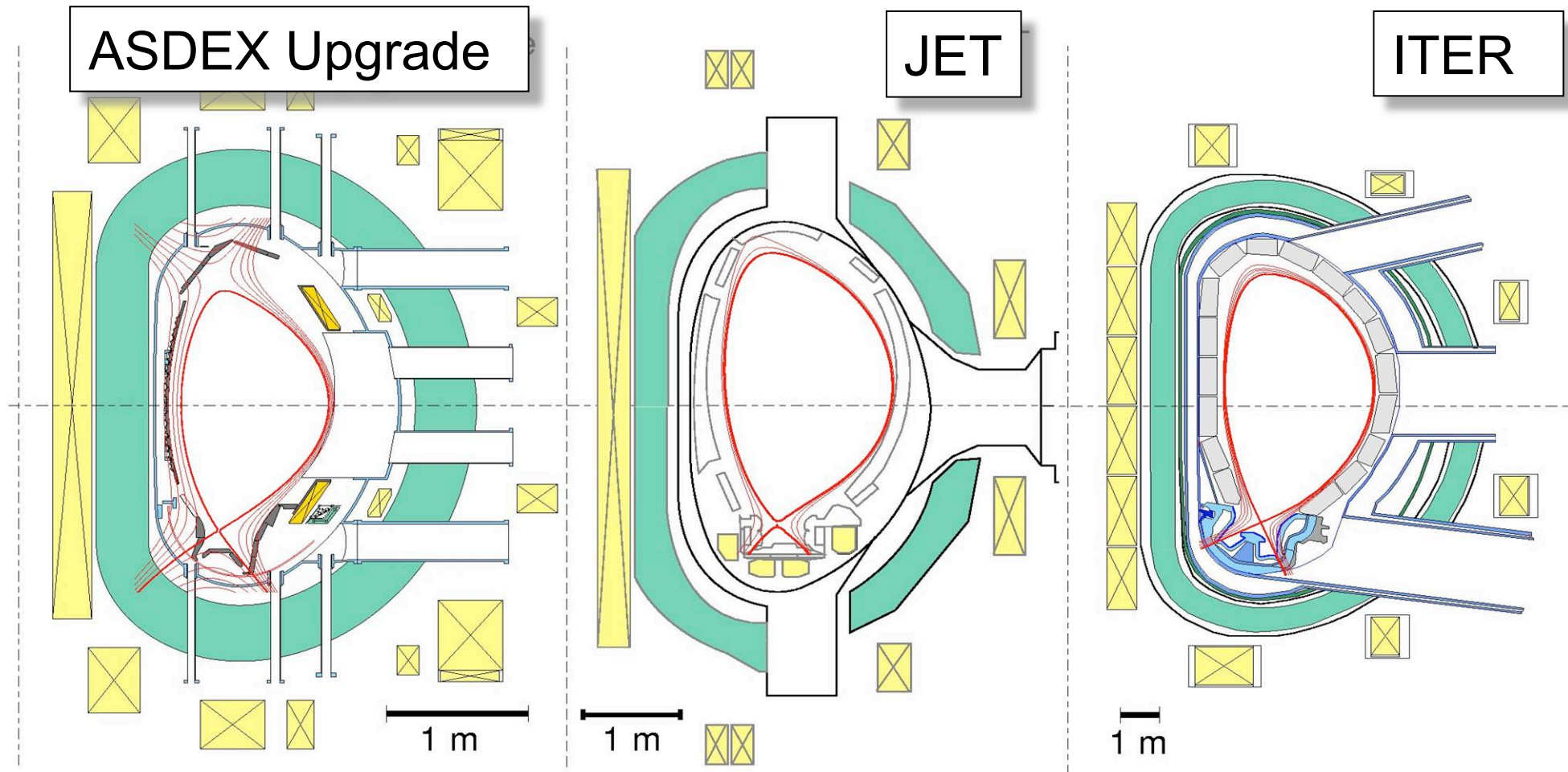
magnetic confinement



inertial confinement

## **fuel density, ion temperature, energy confinement time**

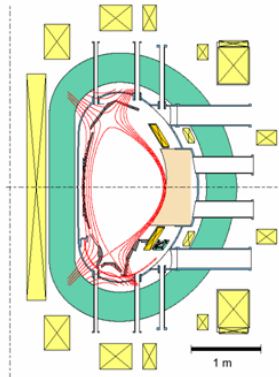
scattering between two nucleons is significantly more probable than fusion fusion for economic energy fusion must be realised in a thermalized plasma  $T = 10 \dots 100 \text{ keV}$  ( $10^8 \dots 10^9 \text{ K}$ )



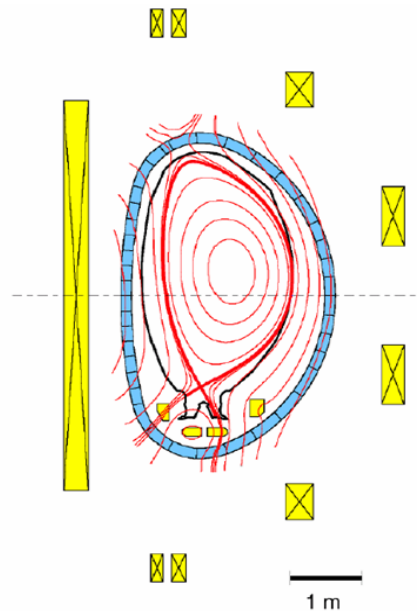
geometry similar to ITER, linear dimensions scale 1:2:4

increase in  $nT\tau$  achievable by

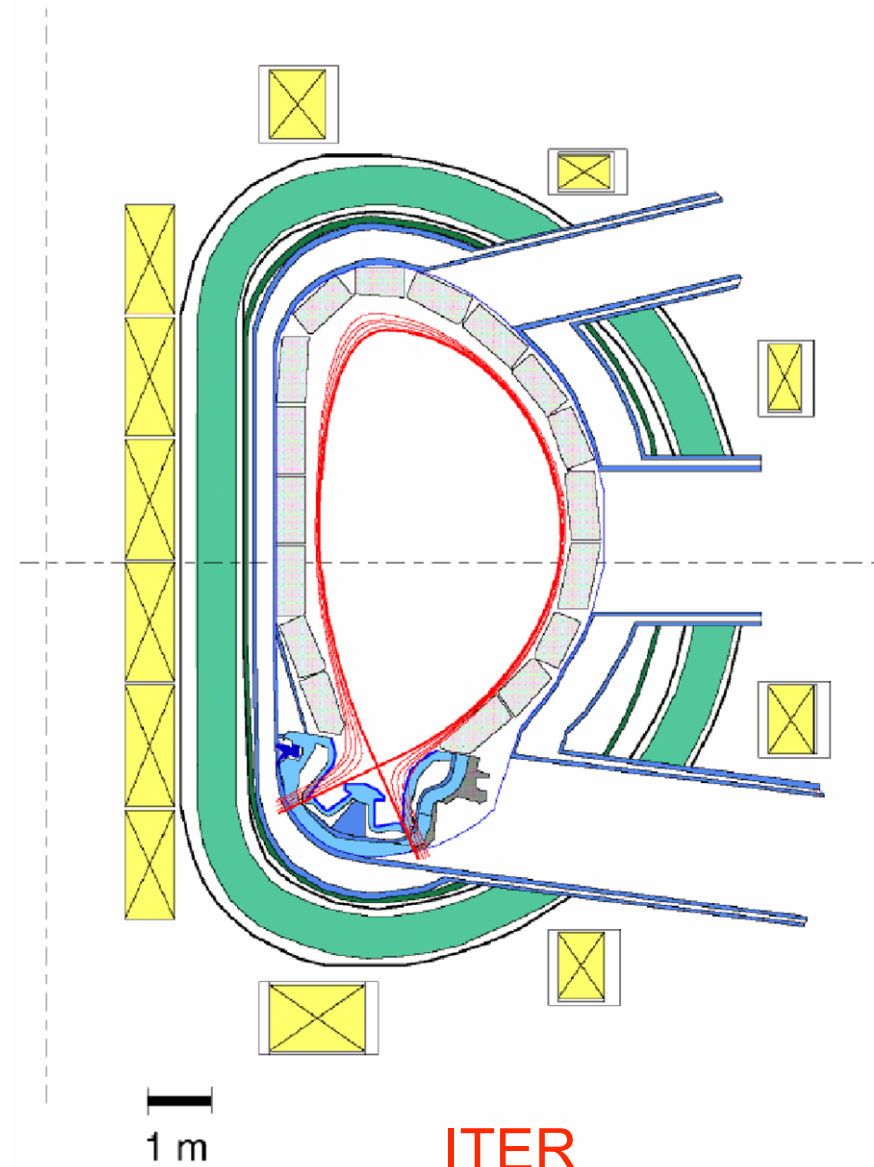
- size
- optimization of stability and confinement



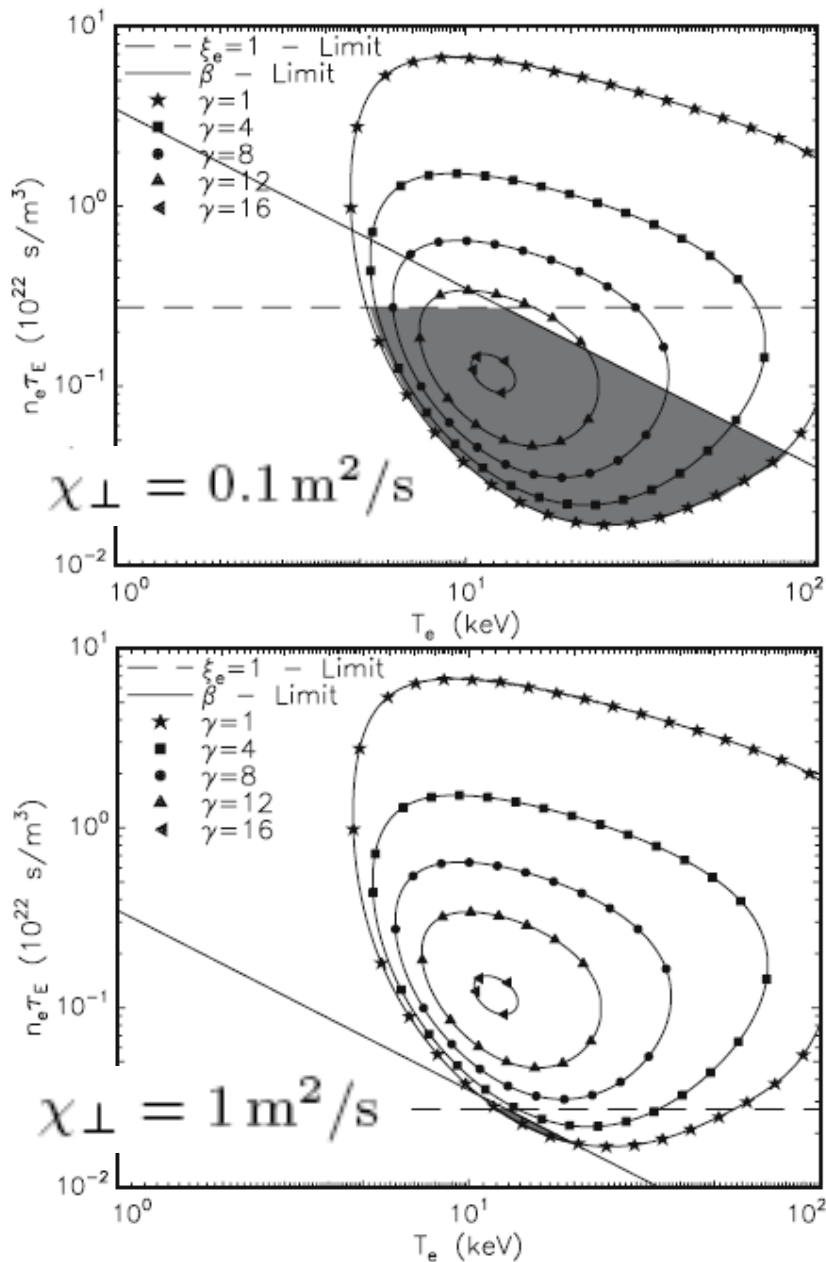
ASDEX-Upgrade



JET



ITER



strong limitation due to  $\beta$ -limit

$$\beta = \frac{\langle p \rangle}{B^2 / (2\mu_0)}$$

allows for a presentation in terms of the density using  $p = 2n_e k_B T_e$  ( $n_e = n_i$ ,  $T_e = T_i$ )

$$n_e \leq \beta^{\text{crit}} \frac{1}{4\mu_0 k_B T_e} B^2$$

multiplying with the energy confinement time  $\tau_E = a^2 / 4\chi_{\perp}$ , we obtain

$$n_e \cdot \tau_E \leq \beta^{\text{crit}} \frac{1}{4\mu_0 k_B T_e} \frac{B^2 a_t^2}{4\chi_{\perp}}$$

and

$$n_e \cdot k_B T_e \cdot \tau_E = \frac{\beta^{\text{crit}} B^2}{16\mu_0} \frac{a_t^2}{\chi_{\perp}}$$

results in restriction of operation space for a  $\chi_{\perp}$

ignition and burn criteria

**particle handling**

power handling

critical impurity concentration

lifetime of wall and divertor elements

choice of material

outlook



In order to maintain a constant plasma density, a fueling flux is required to compensate the losses due to burn up, deposition into wall components and pumping. The global balance equations for the plasma fuel ions (D and T) are

$$\begin{aligned} \frac{dN_D}{dt} &= Q_D - \frac{N_D}{\tau_p} + \frac{N_D}{\tau_p} (1 - f_{\text{wall}} - f_{\text{pump}}) - \frac{N_D N_T}{V_{\text{plasma}}} \langle \sigma v \rangle_{DT} \\ \frac{dN_T}{dt} &= Q_T - \frac{N_T}{\tau_p} + \frac{N_T}{\tau_p} (1 - f_{\text{wall}} - f_{\text{pump}}) - \frac{N_D N_T}{V_{\text{plasma}}} \langle \sigma v \rangle_{DT} \end{aligned}$$

The reservoir in the surface layer with a thickness of about  $0.1 \mu\text{m}$  (taking graphite with its atomic density  $n_C$  and a ratio of hydrogen to carbon of  $c = 0.4$  amounts to

$$\begin{aligned} N_{\text{surface}} &= c n_C S_{\text{divertor}} d \\ &\approx 0.4 \cdot 11.3 \times 10^{28} \text{m}^{-3} \cdot 100 \text{m}^2 \cdot 10^{-7} \text{m} = 4.5 \times 10^{23} \end{aligned}$$

$$N_{\text{plasma}} = n_{\text{plasma}} V_{\text{plasma}} \approx 10^{20} \text{m}^{-3} \cdot 800 \text{m}^3 = 8 \times 10^{22}$$

efficient control of the plasma density is only possible by establishing a strong sink using a pump system and having a source of fuel gas (gas puff, injection of pellets of frozen hydrogen)

Assuming zero recycling, i.e.  $f_{\text{wall}} + f_{\text{pump}} = 1$ , the required inflow rate is (remember the denotation  $n_{\text{D}} = n_{\text{T}} = n_{\text{DT}} = N_{\text{DT}}/V_{\text{plasma}}$ )

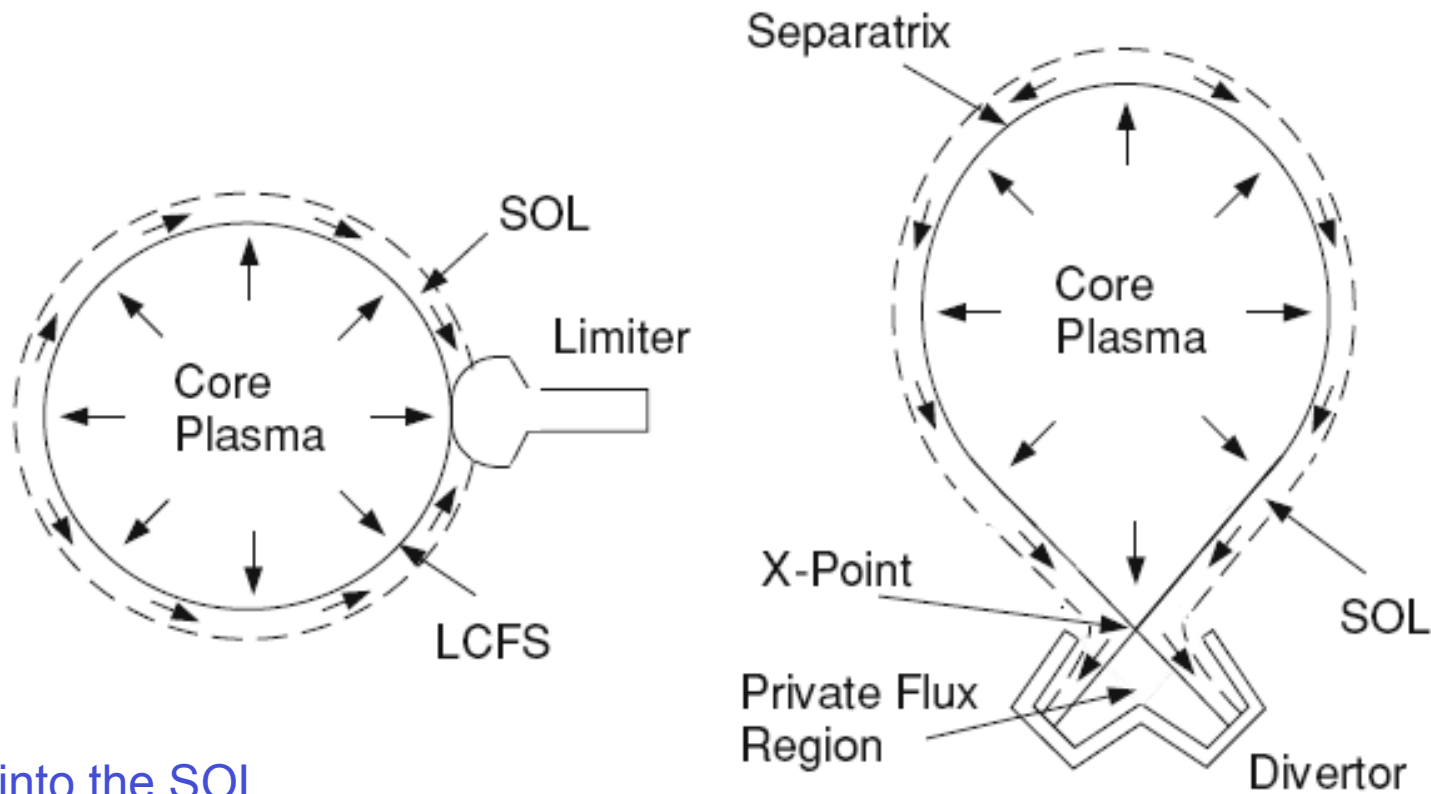
$$Q_{\text{DT}} = Q_{\text{D}} + Q_{\text{T}} = 2 \left[ \frac{n_{\text{DT}}}{\tau_{\text{p}}} + n_{\text{DT}}^2 \langle \sigma v \rangle_{\text{DT}} \right] V_{\text{plasma}}$$

The inflow rate by pellet injection can be estimated [5] as

$$Q_{\text{DT}}^{\text{pel}} = n_e S_{\text{pl}} (a_t - r_{\text{pel}}) / \tau_{\text{pel}}$$

with the pellet deposition radius  $r_{\text{pel}} \approx 0.8 a_t$  and the pellet retention time  $\tau_{\text{pel}} \approx 0.2 \tau_{\text{E}}$ ,  $a_t$  being the minor radius. Even with large pellets ( $d_{\text{pel}} = 5 \text{ mm}$  with  $6 \times 10^{21}$  atoms), the required pellet frequency (in the range of several Hz) in order to ensure  $Q_{\text{DT}}^{\text{pel}} = Q_{\text{DT}}$  will be much larger than the pellet retention time.

besides the plasma density control, the control of plasma composition according to the burning requirements in the main plasma is an additional problem



cross-field flux into the SOL

$$D_{\perp} \frac{dn}{dr} \Big|_{\text{LCFS}} L_c l \approx D_{\perp} \frac{n_{\text{LCFS}}}{\lambda_{\text{SOL}}} L_c l$$

particle flux at the wall

$$l \int_{\text{LCFS}}^{\text{wall}} n_e(r) c_s(r) dr \approx 0.5 n_{\text{LCFS}} c_s \lambda_{\text{SOL}} l$$

plasma decay length

$$\lambda_{\text{SOL}} = \sqrt{\frac{2D_{\perp} L_c}{c_s}} = \sqrt{2D_{\perp} \tau_{\text{SOL}}}$$

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If half of the helium fusion power is transferred into radiation, then the power load onto the wall is given by (see Eq.(10))

$$\begin{aligned}
 P_{\text{rad}}^{\text{wall}} &= 0.5n_{\text{DT}}^2 \langle \sigma v \rangle_{\text{DT}} E_{\alpha} V_{\text{pl}} \approx \frac{n_e^2}{8} \langle \sigma v \rangle_{\text{DT}} E_{\alpha} V_{\text{pl}} \\
 &= \frac{1}{8} (2 \times 10^{20})^2 \times 1.1 \times 10^{-24} 12^2 \times 3.52 \times 10^6 \times 1.6 \times 10^{-19} \times 490 \\
 &= 220 \text{ MW}
 \end{aligned}$$

The resulting heat load is given by  $P_{\text{rad}}^{\text{wall}}/S_{\text{pl}} = P_{\text{rad}}^{\text{wall}}/(4\pi^2 a_t R_t) = 220/490 = 0.45 \text{ MW/m}^2$ .

Due to transport processes, a part of the generated power, namely  $(W_E/\tau_E)V_{\text{pl}}$ , crosses the separatrix into the scrape-off-layer (SOL) (see (9),  $V_{\text{pl}} = 2\pi^2 a_t^2 R_t$ ). In this relatively thin layer around the core plasma, most of this power should be radiated in order to protect the divertor (or limiter) elements against excessive heat loads

$$P_{\text{rad}}^{\text{SOL}} = n_e n_{\text{imp}} L_{\text{imp}} V_{\text{SOL}} \simeq n_e n_{\text{imp}} L_{\text{imp}} 2\pi^2 R_t \lambda_q (\lambda_q + 2a_t)$$

Taking the mentioned  $10 \text{ MW/m}^2$  as the ultimate (technical) limit, the required radiation losses in the SOL can be estimated as

$$P_{\text{rad}}^{\text{SOL}} = \frac{W_{\text{E}} V_{\text{pl}}}{\tau_{\text{E}}} - 10 \text{ MW m}^{-2} \times 2 \times f_{\text{fe}} \times \lambda_{\text{q}} \times 2\pi R_{\text{t}}$$

with the wetted divertor area  $S_{\text{w}} = 2 f_{\text{fe}} \lambda_{\text{q}} 2\pi R_{\text{t}}$ ;  $f_{\text{fe}}$  is the flux expansion factor  $f_{\text{fe}}$ .

let us take a fusion triple product of

$n_{\text{e}} k_{\text{B}} T_{\text{DT}} \tau_{\text{E}} = 1.4 \times 10^6 \text{ kg/(m s)}$  (ITER parameters, see above), and a energy confinement time of about  $\tau_{\text{E}} \simeq a_{\text{t}}^2 / 4 \chi_{\perp} = 2^2 / (4 \times 0.2) = 5 \text{ s}$ .

$$\begin{aligned} P_{\text{rad}}^{\text{SOL}} &= \frac{(3/2) 2 \times 1.4 \times 10^6 \frac{\text{kg}}{\text{ms}} \times V_{\text{pl}}}{(5\text{s})^2} - 10 \text{ MW m}^{-2} \times 2 \times 10 \times 0.004 \text{ m} \times 2\pi 6.2 \text{ m} \\ &= 82.3 - 31.2 \simeq 50 \text{ MW} \end{aligned}$$

since  $W_{\text{E}} \simeq (3/2) 2 n_{\text{e}} k_{\text{B}} T_{\text{DT}} = (3/2) 2 n_{\text{e}} k_{\text{B}} T_{\text{DT}} \tau_{\text{E}} / \tau_{\text{E}}$  and  $V_{\text{pl}} = 2\pi^2 a_{\text{t}}^2 R_{\text{t}} = 2\pi^2 2^2 6.2 = 490 \text{ m}^3$ . The corresponding heat load onto the wall is  $P_{\text{rad}}^{\text{SOL}} / S_{\text{pl}} = 50 / 490 = 0.1 \text{ MW/m}^2$ .

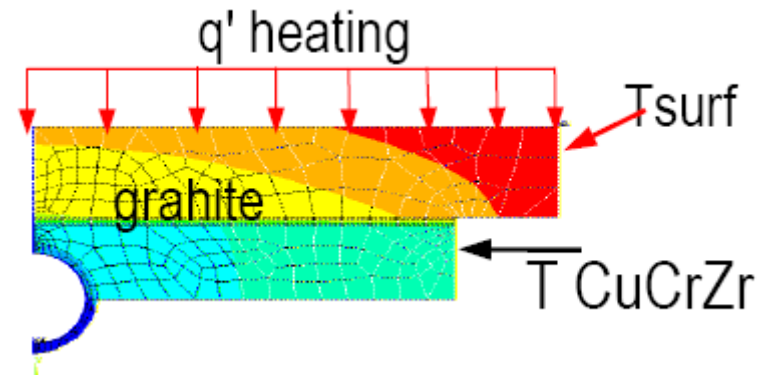
With an average radiation function of  $L_{\text{imp}} \simeq 10^{-32} \text{ Wm}^3$ , a density  $n_e \simeq 5 \times 10^{19} \text{ m}^{-3}$ , and a volume of the SOL of about  $V_{\text{SOL}} = 2\pi^2 R_t \lambda_q (\lambda_q + 2a_t) = 2\pi^2 6.2 \times 0.004 (0.004 + 2 \times 2) = 2 \text{ m}^3$  we obtain a required impurity concentration in the SOL of

$$\frac{n_{\text{imp}}^{\text{SOL}}}{n_e} = \frac{P_{\text{rad}}^{\text{SOL}}}{n_e^2 L_{\text{imp}} V_{\text{SOL}}} = \frac{50 \times 10^6 \text{ W}}{(5 \times 10^{19} \text{ m}^{-3})^2 \times 10^{-32} \text{ Wm}^3 \times 2 \text{ m}^3} = 1,$$

which is too high. By extending the radiation zone at least to some centimeters, e.g.  $\lambda_q \approx 4 \text{ cm}$ , this ratio decreases to a tolerable level of  $n_{\text{imp}}^{\text{SOL}}/n_e = 0.1$  in the SOL.

change of surface temperature

$$P_s = k_{\text{eff}}(T_s - T_{\text{bulk}}) + \varepsilon_g \sigma_{\text{SB}} T_s^4 + \Gamma_{\text{subl}} E_{\text{subl}}$$



heat conduction equation

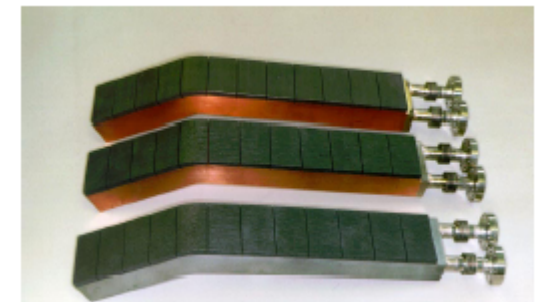
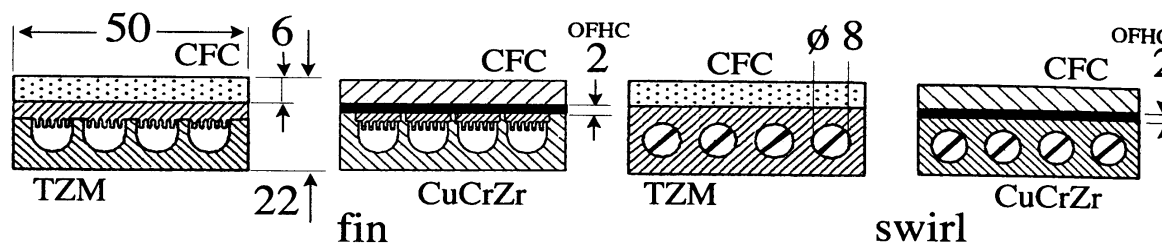
$$c_p \rho \frac{\partial T}{\partial t} = \text{div}(k \text{ grad} T) + Q_E$$

thermal stress criterion

$$\gamma_\sigma = \frac{\sigma}{\sigma_y} = \frac{\alpha_T E_Y P d}{(1 - \nu_P) k \sigma_y} < 1$$

$$\int_0^d P dx = \int_{T_d}^{T_0} k(T) dT \rightarrow P_{\text{max}} = \frac{1}{d} \int_{T_d}^{T_0^{\text{max}}} k(T) dT = \frac{\Lambda(T_0^{\text{max}}) - \Lambda(T_d)}{d}$$

the heat removal capacity is limited by the thermal conductivity of the material



10 MW/m<sup>2</sup> is a challenge



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radiation losses by recombination, bremsstrahlung, cyclotron radiation, and line radiation

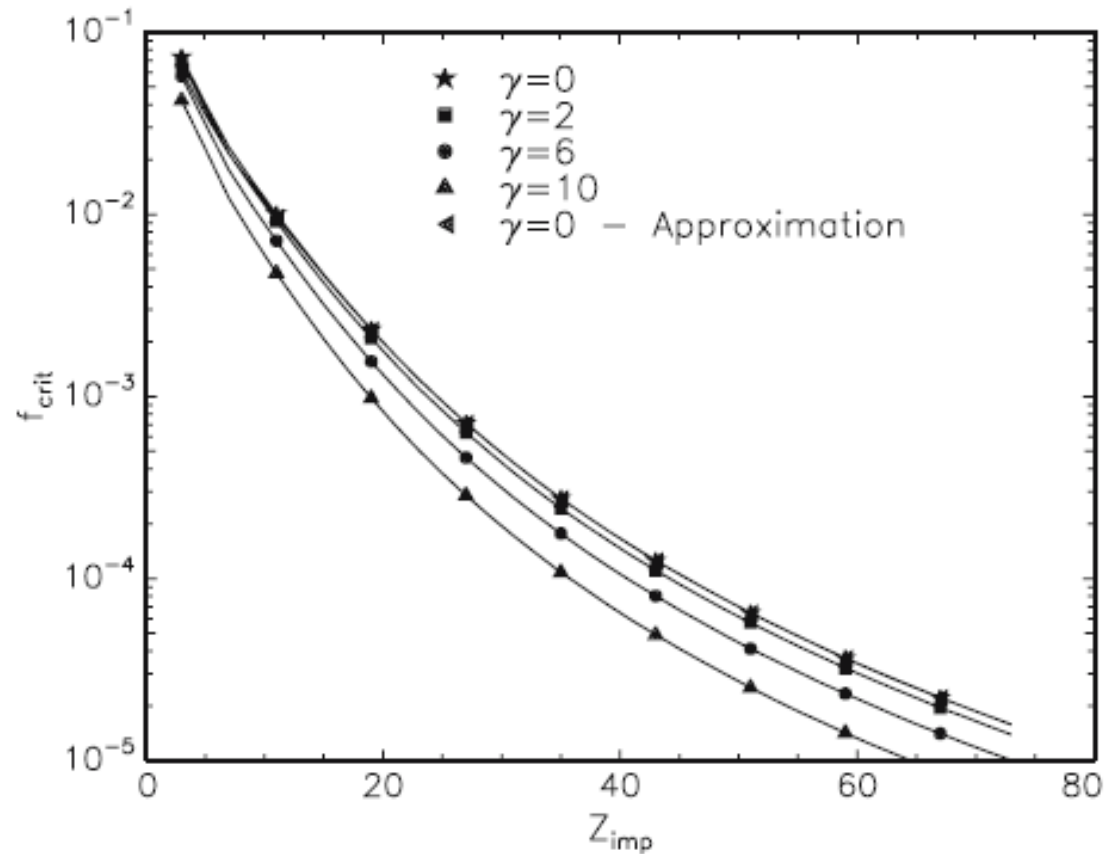
$$P_v = \sum_q [P_{\text{recomb}} + P_{\text{brems}} + P_{\text{cycl}} + P_q]$$

critical impurity concentration as a function of atomic number  $Z_{\text{imp}}$  for  $kT = 12$  keV and different  $\gamma$  as indicated

For given

$$\gamma = \tau_{\text{He}}^* / \tau_E$$

and temperature



approximation

maximum allowable impurity concentration (for 12 keV)

$$f_{\text{crit}} = \frac{\langle \sigma v \rangle_{\text{DT}} E_\alpha (1 - 2f_{\text{He}})^2 - 4c_{\text{br}} \sqrt{k_B T} (1 + 2f_{\text{He}})}{2\langle \sigma v \rangle_{\text{DT}} E_\alpha (1 - 2f_{\text{He}}) q_{\text{imp}} + 4c_{\text{br}} \sqrt{k_B T} (q_{\text{imp}}^2 - q_{\text{imp}}) + 4L_{\text{imp}}}$$

For particle balance, the radial outflux of plasma ions should be equal to the influx of neutrals from the surfaces of the wall, including limiter and divertor plates

$$\begin{aligned} \Gamma_{\text{ions,out}} &= \Gamma_{\text{neutrals,in}} \\ \frac{n_{\text{H}^+} V_{\text{pl}}}{\tau_{\text{p}} S_{\text{pl}}} &= \frac{1}{2} n_{\text{H}_2} v_{\text{H}_2} = n_{\text{H}} v_{\text{H}} \end{aligned}$$

Molecules  $\text{H}_2$  emitted with thermal energies ( $< 0.05$  eV) from surfaces or injected through gas valves undergo predominantly Franck–Condon dissociation processes producing atoms with energies of  $\sim 3$  eV.

Less than half of the produced atoms become ionized by electron impact ionization, while the majority transfers their electrons to the plasma ions in charge-exchange processes.

The flux density of particles sputtered from the wall can be estimated as

$$\Gamma_{\text{ero}} = \frac{n_e V_{\text{pl}}}{\tau_{\text{p}} S_{\text{pl}}} Y_{\text{H}^0} f_{\text{cx}}$$

diffusion equation for a certain ionization stage

$$\frac{\partial n_{\text{imp}}^q}{\partial t} = \text{div}(D_{\perp} \text{grad } n_{\text{imp}}^q) + n_e [n_{\text{imp}}^{q-1} S_{q-1} + n_{\text{imp}}^{q+1} \alpha_{q+1} - n_{\text{imp}}^q (S_q + \alpha_q)] = 0$$

The influx of neutral impurities at a velocity  $v$  penetrating into the plasma is reduced by ionization, and enhanced by recombination

$$\text{div}(n_{\text{imp}}^{q=0} v) = -n_e n_{\text{imp}}^{q=1} S_1 + n_e n_{\text{imp}}^{q=2} \alpha_1$$

The sum for all ionization stages gives the steady state condition that the influx of neutrals should be balanced by diffusion losses

$$D_{\perp} (\partial n_{\text{imp}} / \partial r) = n_{\text{imp}}^{q=0} v = \Gamma_{\text{ero}} \quad n_{\text{imp}} = \sum_{q=1}^{q_{\text{max}}} n_{\text{imp}}^q$$

Integration yields  $n_{\text{tot}}(r) = \frac{v}{D_{\perp}} \int_0^r n_{\text{imp}}^{q=0} dr + n_{\text{imp}}(0)$

with  $n_{\text{imp}}(a_t) = 0$ , the central impurity density becomes simply

$$n_{\text{imp}}(0) = -\frac{v}{D_{\perp}} \int_0^r n_{\text{imp}}^{q=0} dr \simeq \frac{v n_{\text{imp}}^{q=0} \lambda_{\text{iz}}}{D_{\perp}} = \frac{\Gamma_{\text{ero}} \lambda_{\text{iz}}}{D_{\perp}} = \frac{\Phi^{\circ} \lambda_{\text{iz}}}{S_{\text{pl}} D_{\perp}}$$

The impurity concentration is related to the influx of impurities emitted from the wall

$$n_{\text{imp}}(r = 0) = f_{\text{imp}} n_e = \frac{\Gamma_{\text{ero}} \lambda_{\text{iz}}}{D_{\perp}} = \frac{n_e V_{\text{pl}} Y_{\text{H}^0} f_{\text{cx}} \lambda_{\text{iz}}}{D_{\perp} \tau_{\text{p}} S_{\text{pl}}}$$

and substituting  $\tau_{\text{p}} \simeq a_{\text{t}}^2 / (4D_{\perp})$  gives

$$V_{\text{pl}} = 2\pi^2 a_{\text{t}}^2 R_{\text{t}}$$

$$S_{\text{pl}} = 4\pi^2 a_{\text{t}} R_{\text{t}}$$

$$f_{\text{imp}} = Y_{\text{H}^0} f_{\text{cx}} \frac{2\lambda_{\text{iz}}}{a_{\text{t}}}$$

For a burning fusion plasma, the ratio  $f_{\text{imp}} / f_{\text{crit}}$

$$\gamma_{\text{imp}} = \frac{f_{\text{imp}}}{f_{\text{crit}}} = Y_{\text{H}^0} f_{\text{cx}} \frac{2\lambda_{\text{iz}}}{a_{\text{t}}} \times \frac{2\langle\sigma v\rangle_{\text{DT}} E_{\alpha} (1 - 2f_{\text{He}}) q_{\text{imp}} + 4c_{\text{br}} \sqrt{k_{\text{B}} T} (q_{\text{imp}}^2 - q_{\text{imp}}) + 4L_{\text{imp}}}{\langle\sigma v\rangle_{\text{DT}} E_{\alpha} (1 - 2f_{\text{He}})^2 - 4c_{\text{br}} \sqrt{k_{\text{B}} T} (1 + 2f_{\text{He}})} \leq 1$$

should be kept well below unity

The resulting impurity concentration in the core

$$n_{\text{imp}}(0) \simeq \frac{\lambda_{\text{iz}}}{S_{\text{pl}} D_{\perp}} \frac{n_{\text{imp}}^{\text{SOL}} V_{\text{SOL}}}{\tau_{\text{imp}}^{\text{SOL}}} Y_{\text{imp} \rightarrow \text{wall}}$$

could be too high to fulfil the criterion  $n_{\text{imp}}(r = 0)/n_e < f_{\text{crit}}$ , especially with high impurity concentration in the SOL  $n_{\text{imp}}^{\text{SOL}}$ , high sputtering yield (for example  $Y_{\text{Ar} \rightarrow \text{W}}$ ), and short retention times of the impurity ions in the edge plasma  $\tau_{\text{imp}}^{\text{SOL}}$ .

$$\begin{aligned} n_{\text{imp}}(0) &\approx \frac{\lambda_{\text{iz}}}{S_{\text{pl}} D_{\perp}} \frac{P_{\text{rad}}^{\text{SOL}}}{n_e L_{\text{imp}} V_{\text{SOL}}} \frac{V_{\text{SOL}}}{\tau_{\text{imp}}^{\text{SOL}}} Y_{\text{imp} \rightarrow \text{wall}} \\ &= \frac{\lambda_{\text{iz}}}{S_{\text{pl}} D_{\perp}} \frac{W_{\text{E}} V_{\text{pl}}}{\tau_{\text{E}}} \frac{1}{n_e L_{\text{imp}} \tau_{\text{imp}}^{\text{SOL}}} Y_{\text{imp} \rightarrow \text{wall}} \\ &= \frac{\lambda_{\text{iz}}}{D_{\perp}} \frac{(3/2) 2 n_e k_{\text{B}} T_i (a_t/2)}{a_t^2 / (4 \chi_{\perp})} \frac{1}{n_e L_{\text{imp}} \tau_{\text{imp}}^{\text{SOL}}} Y_{\text{imp} \rightarrow \text{wall}} \\ &= \frac{k_{\text{B}} T_i}{a_t} \frac{6 \lambda_{\text{iz}} Y_{\text{imp} \rightarrow \text{wall}}}{\tau_{\text{imp}}^{\text{SOL}} L_{\text{imp}}} \end{aligned}$$

ignition and burn criteria

particle handling

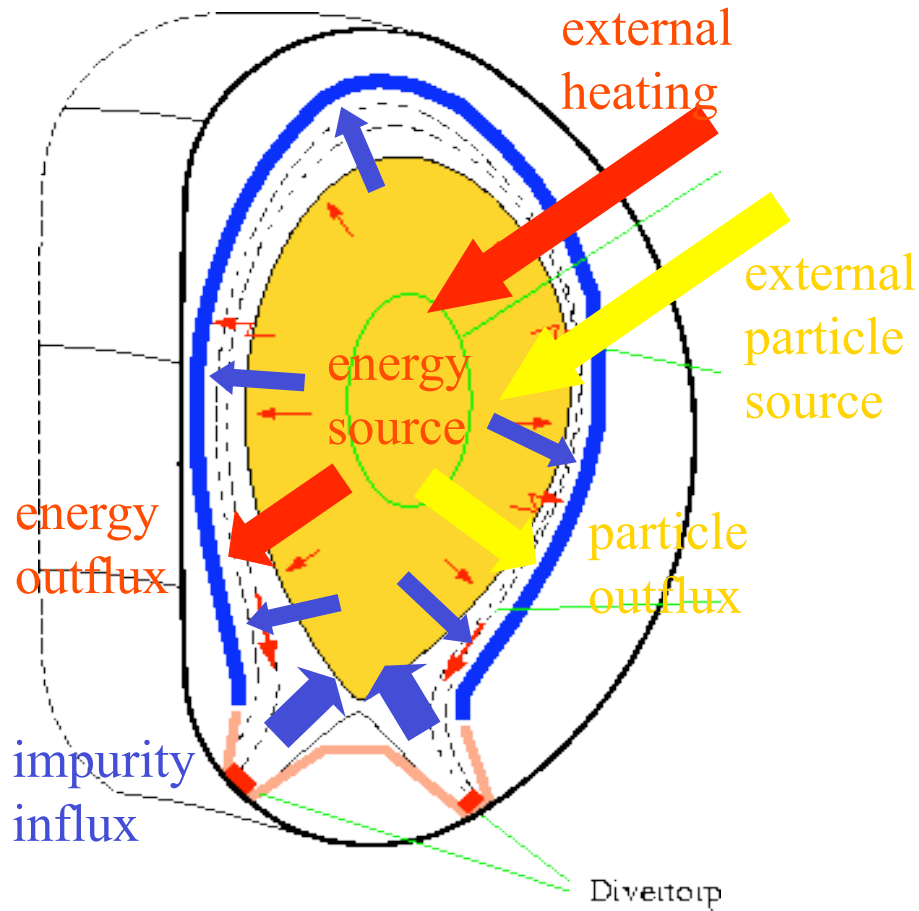
power handling

critical impurity concentration

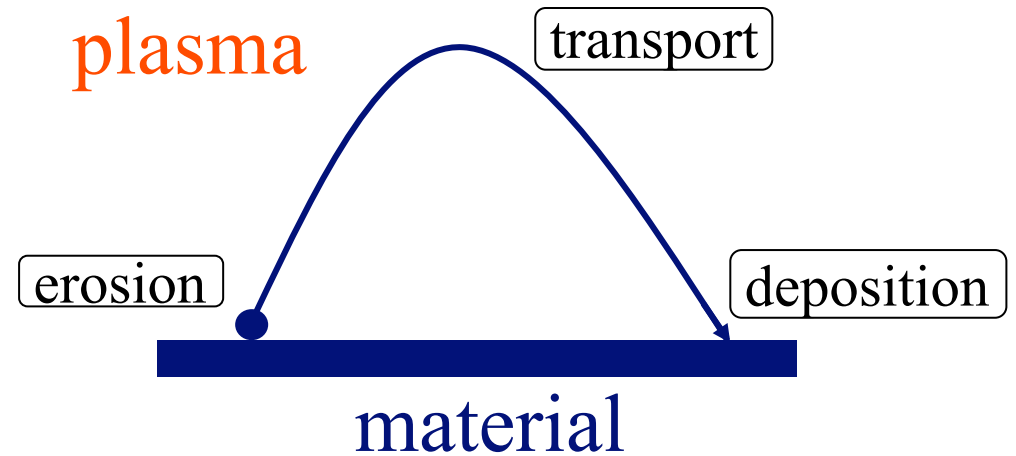
**lifetime of wall and divertor elements**

choice of material

outlook

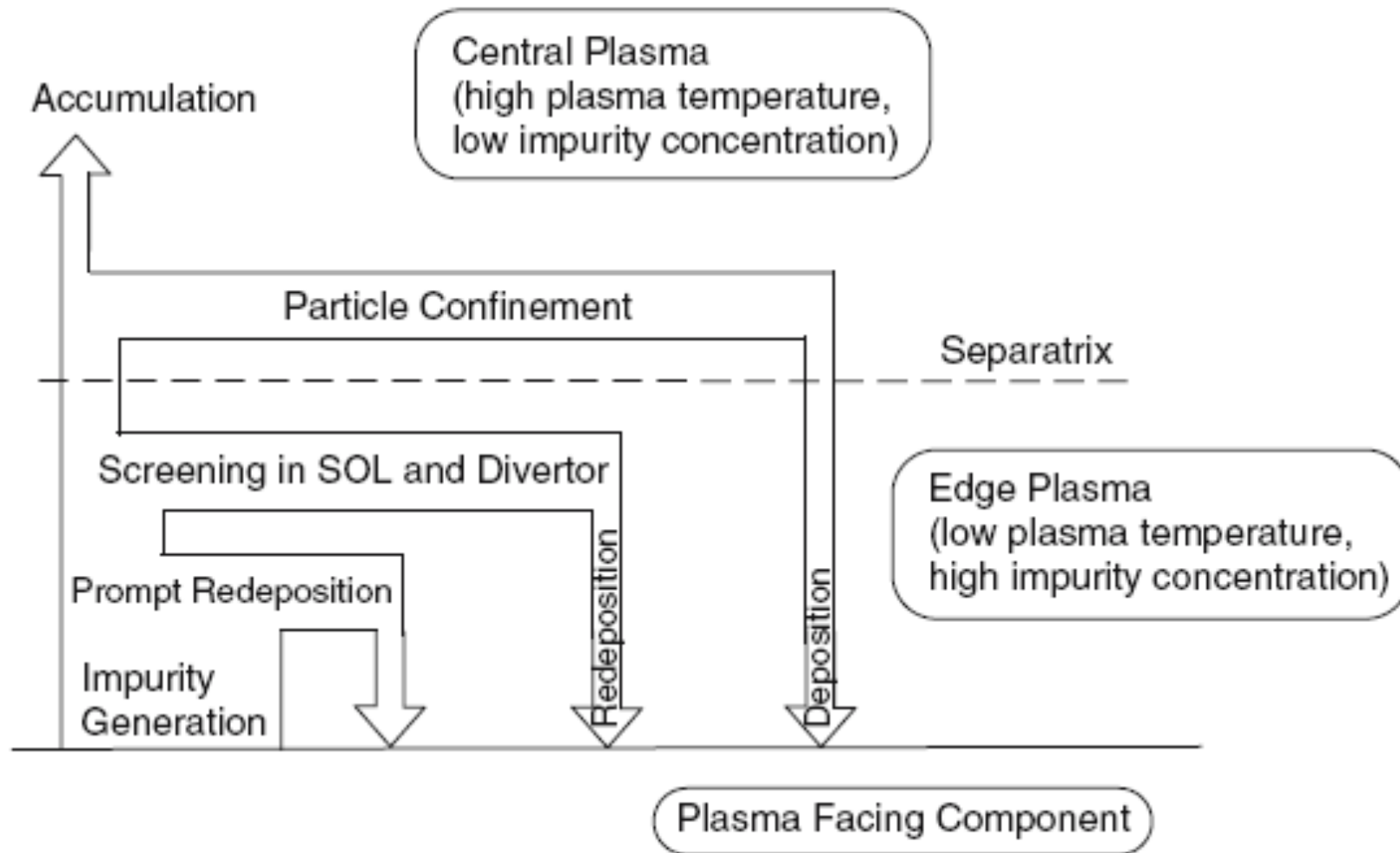


- impurity release and plasma contamination
- distribution of particle emission
- ionization
- prompt redeposition
- transport and collision in the edge plasma
- deposition and redeposition
- sticking



cycle of erosion, transport and deposition



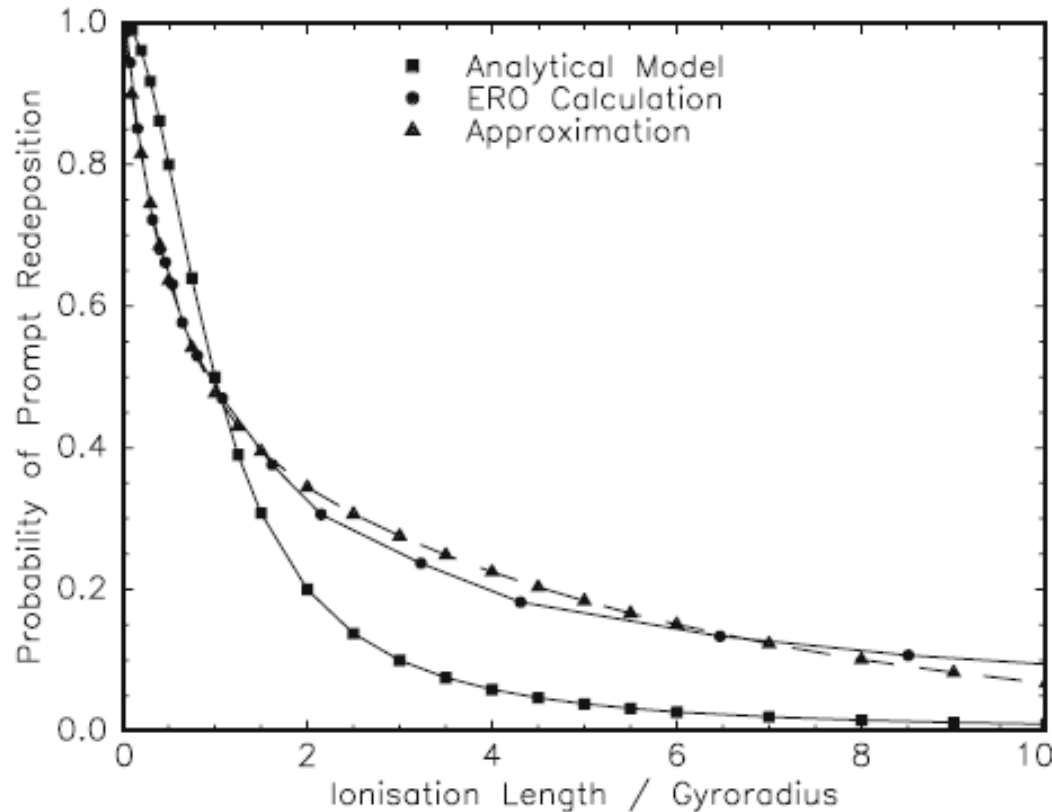


the net effects are orders of magnitude smaller than the gross ones

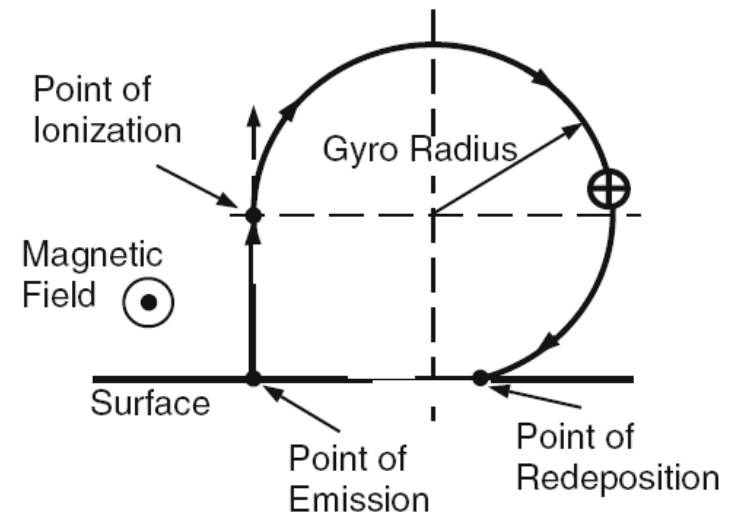
force on an impurity ion along the magnetic field (in the SOL)

$$F_{\text{total}} = -\frac{1}{n} \frac{\partial p}{\partial s} + m \frac{(v_{\text{pl}} - v)}{\tau_s} + q e E + \alpha_e \frac{\partial T_e}{\partial s} + \beta_i \frac{\partial T_i}{\partial s}$$

prompt redeposition



probability of prompt redeposition



$$\gamma_p = \lambda_{iz} / \rho$$

$$P_{\text{pr}} = \frac{1}{1 + \gamma_p^2}$$

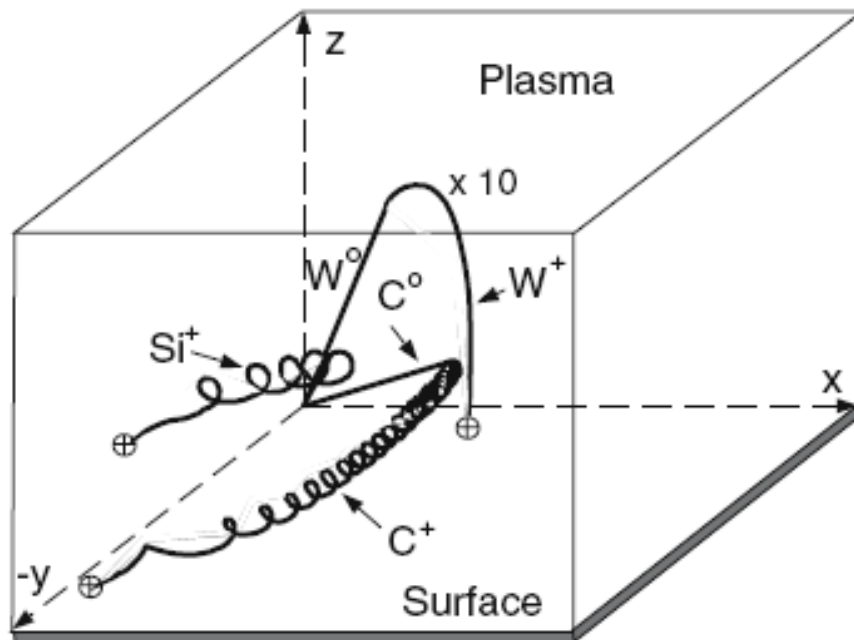
$$P_{\text{pr}} = \frac{1}{2} \left[ \exp(-2\gamma_p) + \exp\left(-\frac{\gamma_p}{5}\right) \right]$$

1. simulation region and background plasma
2. calculation of yields and emission of neutrals
3. transport of neutrals – reaction control
4. transport of impurity ions
5. deposition of ions
6. change of surface composition

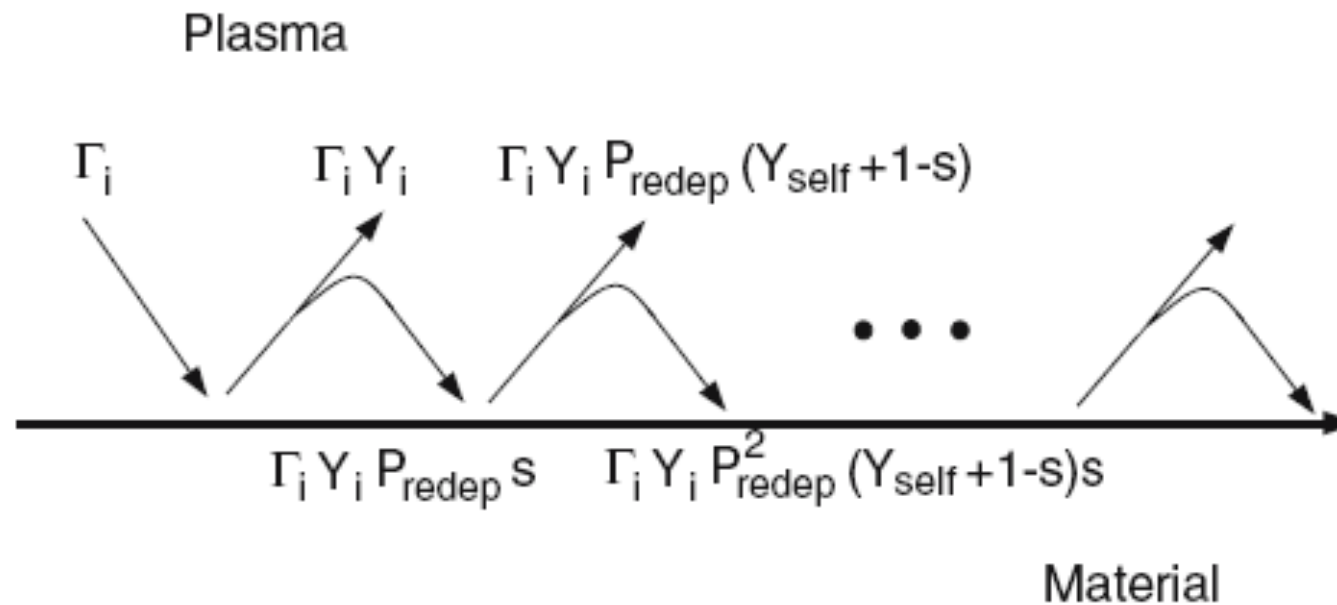
choice of reaction

$$\text{RND} \leq P = 1 - \exp(-\Delta t_2/\tau)$$

$$\tau = \frac{1}{n_e \sum_i \langle \sigma v \rangle_i}$$



$$M \frac{dv}{dt} = F = Q(E + v \times B) + M \left. \frac{\partial v}{\partial t} \right|_{\text{collisions}}$$



$$\Gamma_{\text{ero}}^{\text{gross}} = \Gamma_e \sum_i Y_i f_i \left( 1 + P_{\text{redep}} [Y_{\text{self}} + 1 - s] \right. \\ \left. + (P_{\text{redep}} [Y_{\text{self}} + 1 - s])^2 + \dots \right) \\ = \frac{\Gamma_e \sum_i Y_i f_i}{1 - P_{\text{redep}} [Y_{\text{self}} + 1 - s]}$$

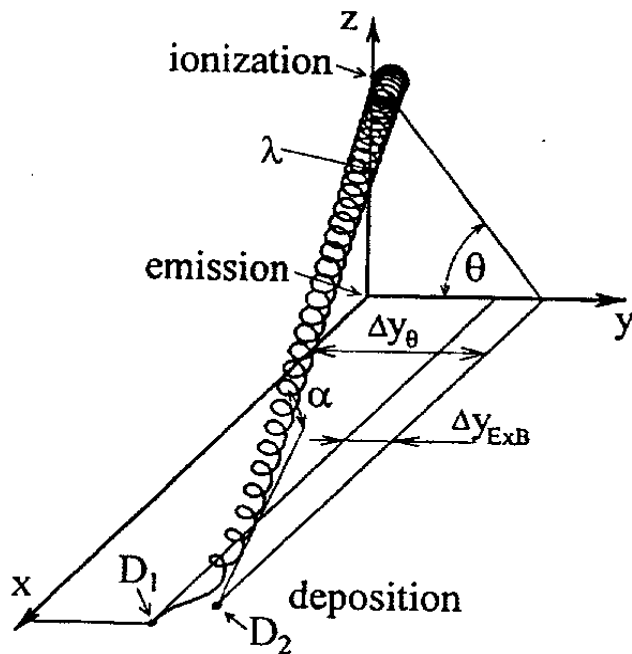
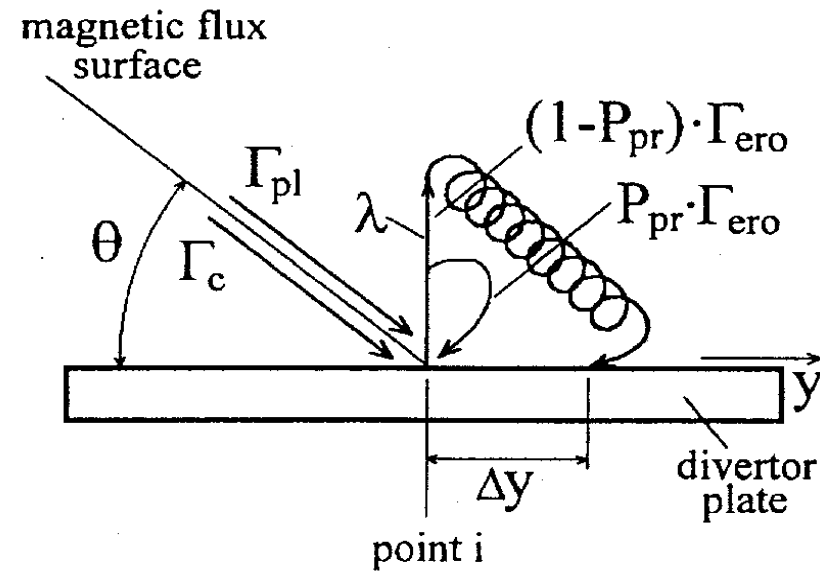
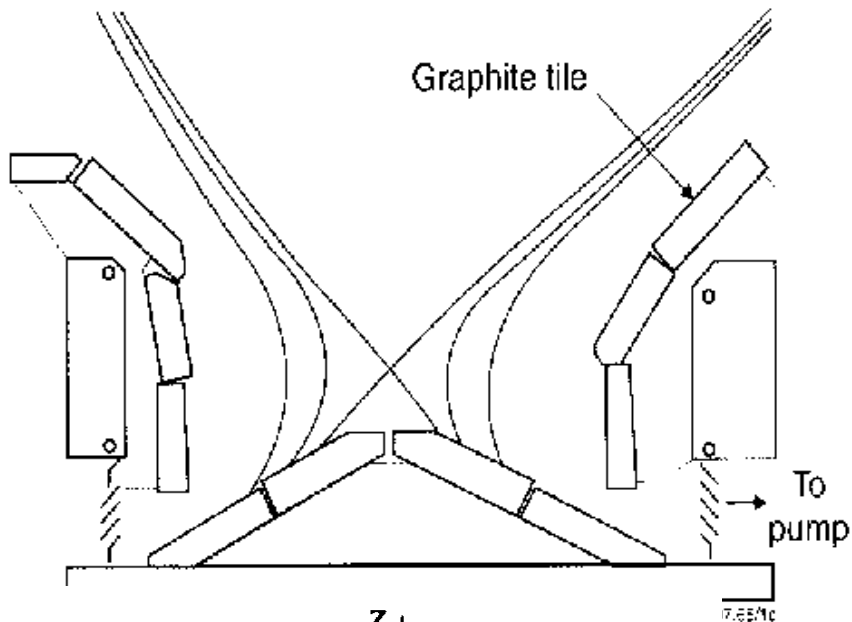
$$\Gamma_{\text{redep}} = s \Gamma_e P_{\text{redep}} \sum_i Y_i f_i \left( 1 + P_{\text{redep}} [Y_{\text{self}} + 1 - s] \right. \\ \left. + (P_{\text{redep}} [Y_{\text{self}} + 1 - s])^2 + \dots \right) \\ = \frac{s \Gamma_e P_{\text{redep}} \sum_i Y_i f_i}{1 - P_{\text{redep}} [Y_{\text{self}} + 1 - s]},$$

$$\begin{aligned}
 \Gamma_{\text{ero}}^{\text{gross}} &= \Gamma_e \sum_i Y_i f_i \left( 1 + P_{\text{redep}} [Y_{\text{self}} + 1 - s] \right. \\
 &\quad \left. + (P_{\text{redep}} [Y_{\text{self}} + 1 - s])^2 + \dots \right) \\
 &= \frac{\Gamma_e \sum_i Y_i f_i}{1 - P_{\text{redep}} [Y_{\text{self}} + 1 - s]} \\
 \Gamma_{\text{redep}} &= s \Gamma_e P_{\text{redep}} \sum_i Y_i f_i \left( 1 + P_{\text{redep}} [Y_{\text{self}} + 1 - s] \right. \\
 &\quad \left. + (P_{\text{redep}} [Y_{\text{self}} + 1 - s])^2 + \dots \right) \\
 &= \frac{s \Gamma_e P_{\text{redep}} \sum_i Y_i f_i}{1 - P_{\text{redep}} [Y_{\text{self}} + 1 - s]},
 \end{aligned}$$

the redeposited material impurities stick to the surface with the probability  $s$

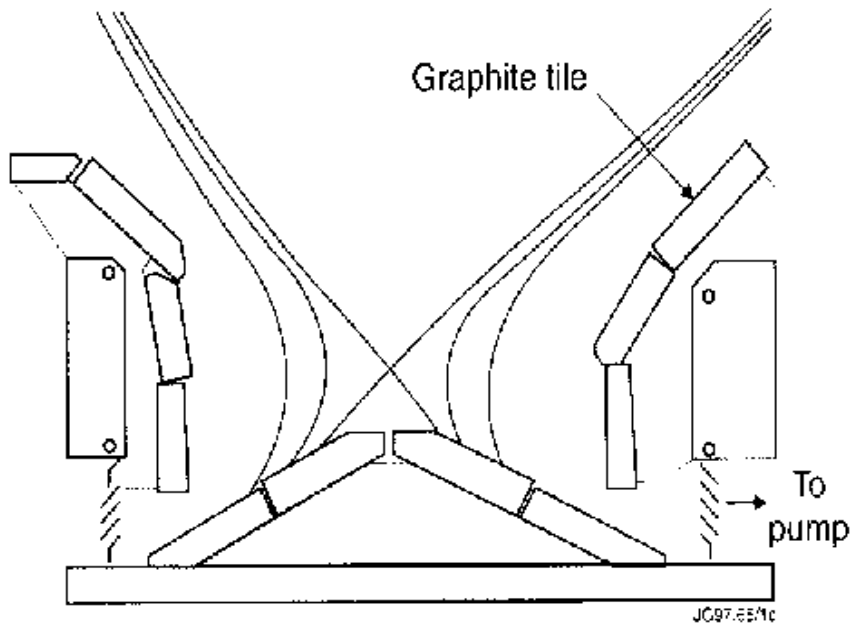
$$\Gamma_{\text{ero}}^{\text{net}} = \Gamma_{\text{ero}}^{\text{gross}} - \Gamma_{\text{redep}}$$

$$\Gamma_{\text{ero}}^{\text{net}} = \Gamma_e \frac{(1 - s P_{\text{redep}}) \sum_i Y_i f_i}{1 - P_{\text{redep}} [Y_{\text{self}} + 1 - s]} = \Gamma_e Y_{\text{eff}}$$

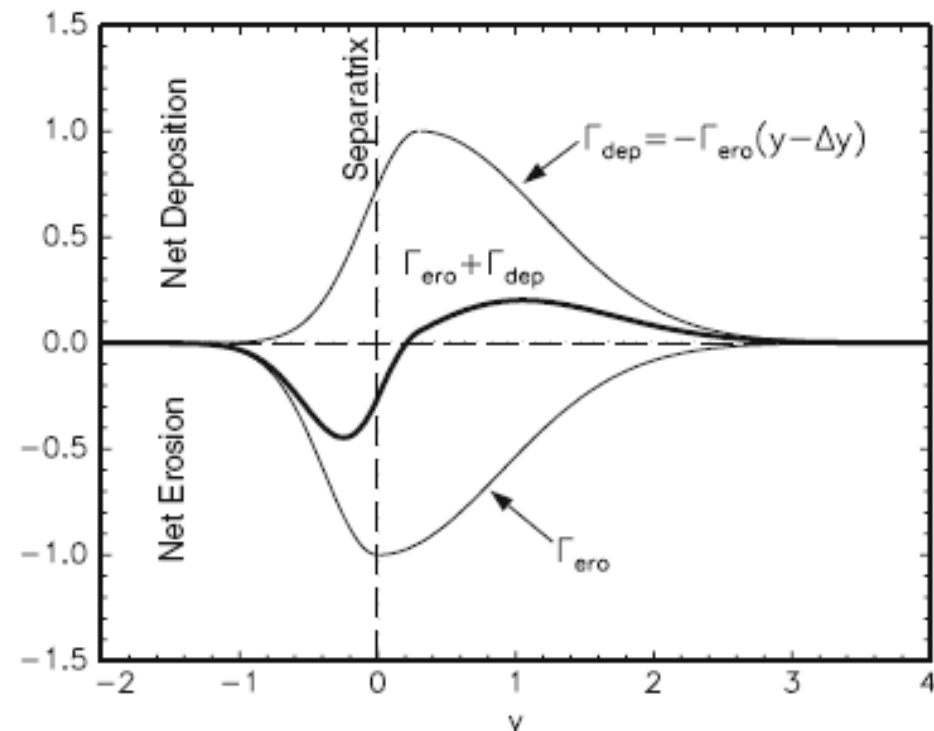


$$\Delta y = \Delta y_\theta + \Delta y_{E \times B} \rightarrow 0$$

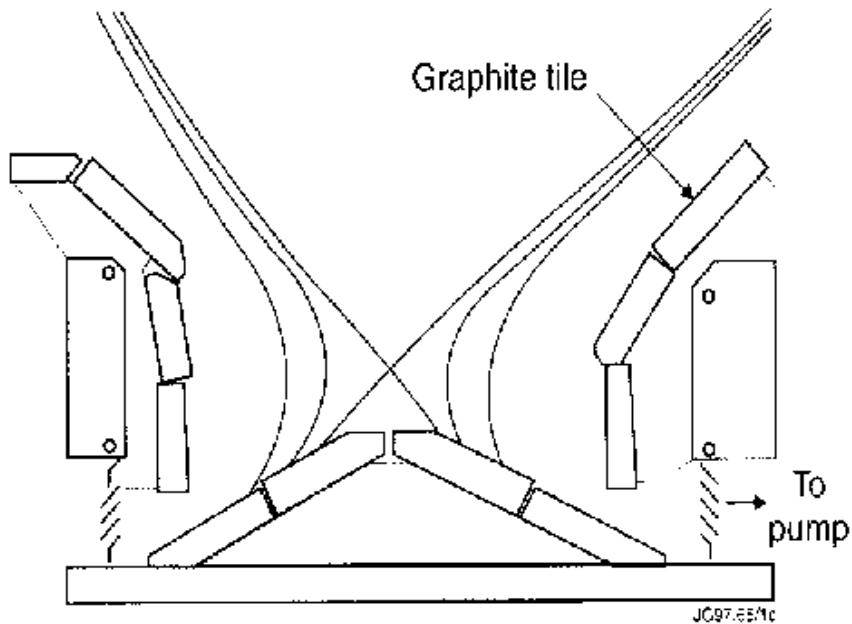
for zero erosion



$$\begin{aligned} \Gamma_{\text{ero/dep}}(y) &= \Gamma_{\text{ero}}(y) + \Gamma_{\text{dep}}(y) \\ &= \Gamma_{\text{ero}} - \Gamma_{\text{ero}}(y - \Delta y) \approx \frac{\partial \Gamma_{\text{ero}}}{\partial y} \Delta y \end{aligned}$$



sample erosion/deposition profile across the divertor plate



$$\Delta y = \Delta y_{\theta} + \Delta y_{E \times B} \rightarrow 0 \quad \text{for zero erosion}$$

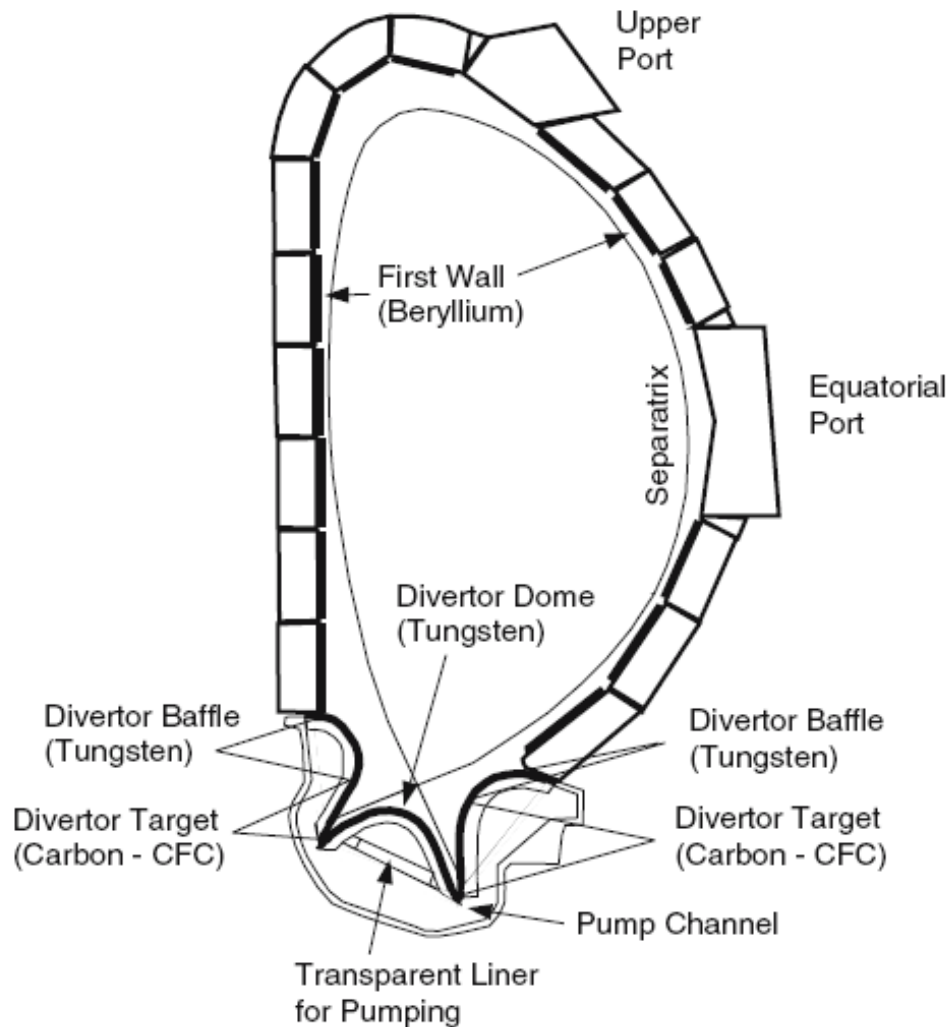
$$\gamma_{\text{ero}} = \frac{\Delta y}{|\lambda_{n,1}|} = \frac{\Delta y_{\theta} + \Delta y_{E \times B}}{|\lambda_{n,1}|}$$

$$\Delta_{\text{ero}}^{\text{divertor}} = \frac{t_{\text{exp}} \Gamma_{\text{pl}} Y_{\text{pl}} (1 - s P_{\text{pr}})}{n_{\text{o}} (1 - P_{\text{pr}} [Y_{\text{self}} + 1 - s])} \tanh(C \gamma_{\text{ero}})$$



at the wall erosion due to cx-neutrals

$$\begin{aligned} \Delta_{\text{ero}}^{\text{wall}} &= \frac{Y \Gamma_{\text{H}^0} t_{\text{discharge}}}{n_{\text{carbon}}} = \frac{Y_{\text{H}^0} n_e V_{\text{pl}} t_{\text{discharge}}}{\tau_p S_{\text{pl}} n_{\text{carbon}}} f_{\text{cx}} \\ &= Y_{\text{H}^0} n_e t_{\text{discharge}} f_{\text{cx}} \frac{2D_{\perp}}{n_{\text{carbon}} a_t} \end{aligned}$$



at the divertor plates due to plasma erosion

$$\gamma_{\text{ero}} = \frac{\Delta y}{|\lambda_{n,1}|} = \frac{\Delta y_{\theta} + \Delta y_{E \times B}}{|\lambda_{n,1}|}$$

$$\Delta_{\text{ero}}^{\text{divertor}} = \frac{t_{\text{exp}} \Gamma_{\text{pl}} Y_{\text{pl}} (1 - s P_{\text{pr}})}{n_o (1 - P_{\text{pr}} [Y_{\text{self}} + 1 - s])} \tanh(C \gamma_{\text{ero}})$$

ignition and burn criteria

particle handling

power handling

critical impurity concentration

lifetime of wall and divertor elements

**choice of material**

outlook

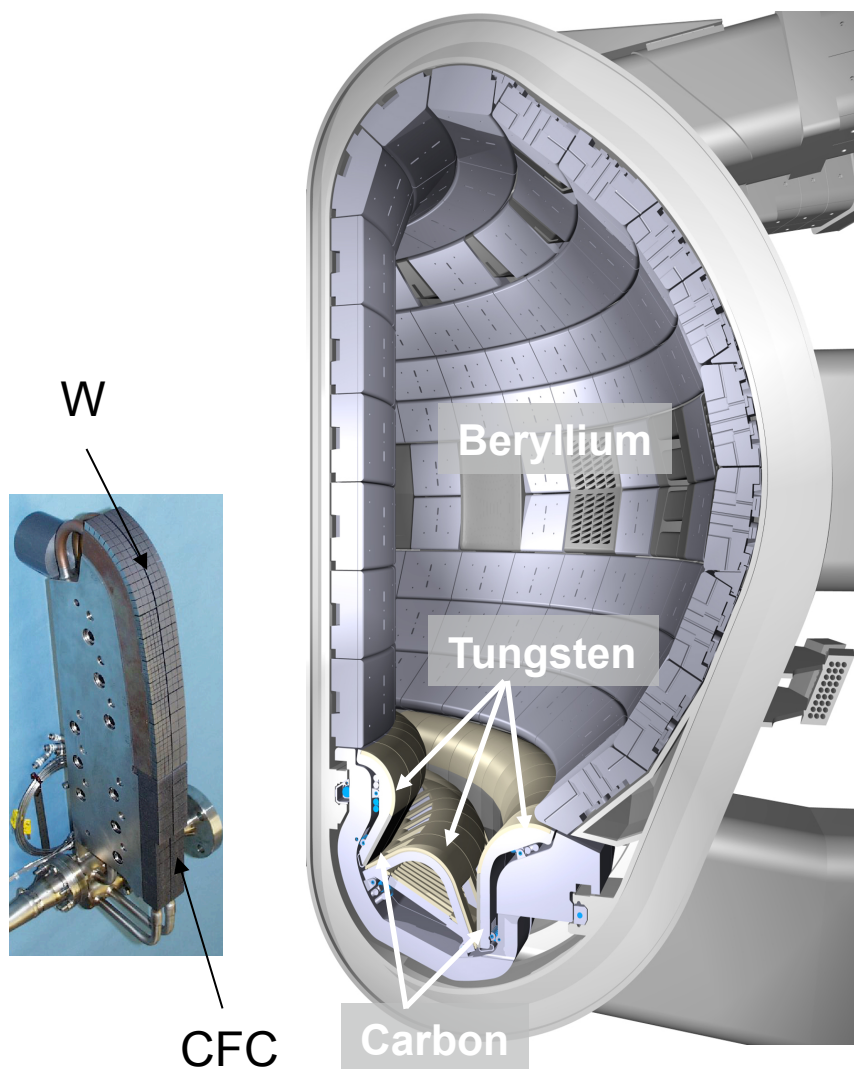
- high heat conductivity and capacity
- high melting point
- large thermal shock resistance
- good handling with respect to machining
- little degradation of the thermonphysical properties
- and low activation and transmutations due to the 14 MeV neutron flux
- low permanent tritium retention
- low erosion due to bombardment with plasma ions and neutrals
- low self-sputtering
- low erosion due to ,local‘ effects (electric arcs, hot spots)
- low energy loss by radiation if the impurity atoms enter the central plasma



Where are the ,candidates‘?

# Choice of Material

	I	II	IIIb	IVb		IIIb	III	IV	V	VI	VII	VIII
1	<b>1</b> <b>H</b> 1,007											<b>2</b> <b>He</b> 4,002
2	<b>3</b> <b>Li</b> 6,941	<b>4</b> <b>Be</b> 9,012					<b>5</b> <b>B</b> 10,81	<b>6</b> <b>C</b>	<b>7</b> <b>N</b> 14,00	<b>8</b> <b>O</b> 15,99	<b>9</b> <b>F</b> 18,99	<b>10</b> <b>Ne</b> 20,17
3	<b>11</b> <b>Na</b> 22,98	<b>12</b> <b>Mg</b> 24,30					<b>13</b> <b>Al</b>	<b>14</b> <b>Si</b>	<b>15</b> <b>P</b>	<b>16</b> <b>S</b>	<b>17</b> <b>Cl</b>	<b>18</b> <b>Ar</b>
4	<b>19</b> <b>K</b> 39,09	<b>20</b> <b>Ca</b> 40,07	<b>21</b> <b>Sc</b> 44,95	<b>22</b> <b>Ti</b> 47,88	<b>23</b> <b>V</b> 50,94	<b>24</b> <b>Cr</b> 51,99	<b>25</b> <b>Mn</b> 54,93	<b>26</b> <b>Fe</b> 55,84	<b>27</b> <b>Co</b> 58,93	<b>28</b> <b>Ni</b> 58,69	<b>29</b> <b>Cu</b> 63,54	<b>30</b> <b>Zn</b> 65,3
5	<b>37</b> <b>Rb</b> 85,46	<b>38</b> <b>Sr</b> 87,62	<b>39</b> <b>Y</b> 88,90	<b>40</b> <b>Zr</b> 91,22	<b>41</b> <b>Nb</b> 92,90	<b>42</b> <b>Mo</b> 95,94	<b>43</b> <b>Tc</b> 98,90	<b>44</b> <b>Ru</b> 101,0	<b>45</b> <b>Rh</b> 102,9	<b>46</b> <b>Pd</b> 106,4	<b>47</b> <b>Ag</b> 107,8	<b>48</b> <b>Cd</b> 112
6	<b>55</b> <b>Cs</b> 132,9	<b>56</b> <b>Ba</b> 137,3		<b>72</b> <b>Hf</b> 178,4	<b>73</b> <b>Ta</b> 180,9	<b>74</b> <b>W</b> 186,2	<b>75</b> <b>Re</b> 186,2	<b>76</b> <b>Os</b> 190,2	<b>77</b> <b>Ir</b> 192,2	<b>78</b> <b>Pt</b> 195,0	<b>79</b> <b>Au</b> 196,9	<b>80</b> <b>Hg</b> 200
7	<b>87</b> <b>Fr</b> 223,0	<b>88</b> <b>Ra</b> 226,0		<b>104</b> <b>Rf</b> 261,1	<b>105</b> <b>Db</b> 262,1	<b>106</b> <b>Sg</b> 263,1	<b>107</b> <b>Bh</b> 262,1	<b>108</b> <b>Hs</b> 263	<b>109</b> <b>Mt</b> 266	<b>110</b> <b>Uun</b> 269	<b>111</b> <b>Uuu</b> 272	<b>112</b> <b>Uu</b> 277
				<b>57</b> <b>La</b> 138,9	<b>58</b> <b>Ce</b> 140,1	<b>59</b> <b>Pr</b> 140,9	<b>60</b> <b>Nd</b> 144,2	<b>61</b> <b>Pm</b> 146,9	<b>62</b> <b>Sm</b> 150,3	<b>63</b> <b>Eu</b> 151,9	<b>64</b> <b>Gd</b> 157,2	<b>65</b> <b>Tb</b> 158
				<b>89</b> <b>Ac</b> 227,0	<b>90</b> <b>Th</b> 232,0	<b>91</b> <b>Pa</b> 231,0	<b>92</b> <b>U</b> 238,0	<b>93</b> <b>Np</b> 237,0	<b>94</b> <b>Pu</b> 244,0	<b>95</b> <b>Am</b> 243,0	<b>96</b> <b>Cm</b> 247,0	<b>97</b> <b>Bk</b> 247,0



**Be:** 700 m<sup>2</sup>, low Z, large getter capability of oxygen and strong hydrogenic pumping (easy density control), but low melting temperature and high sputtering yields

**W:** 100 m<sup>2</sup>, high Z, negligible erosion at low plasma temperatures, but reduced heat load capability (melting), poor thermal shock behavior, blistering, accumulation?

**C:** 50 m<sup>2</sup>, low Z, excellent thermophysical properties, shows no melting, has high thermal shock resistance, but large chemical erosion, codeposition with tritium, flakes and dust

**Problem: Material Mixing**

cross-section of ITER with plasma-facing components including the first wall, the V-shaped divertor slots with the divertor targets, baffles, and the divertor dome

ignition and burn criteria

particle handling

power handling

critical impurity concentration

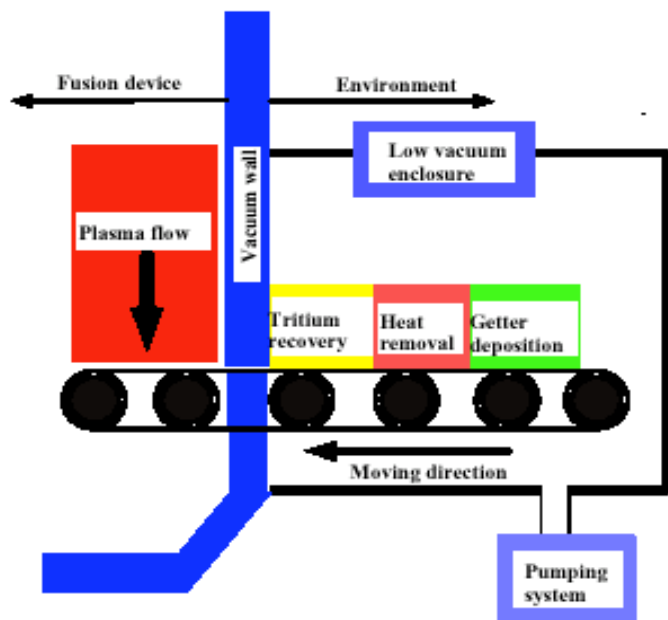
lifetime of wall and divertor elements

choice of material

**outlook**

## moving-belt PFC concept

(Hirooka et al. 1997)



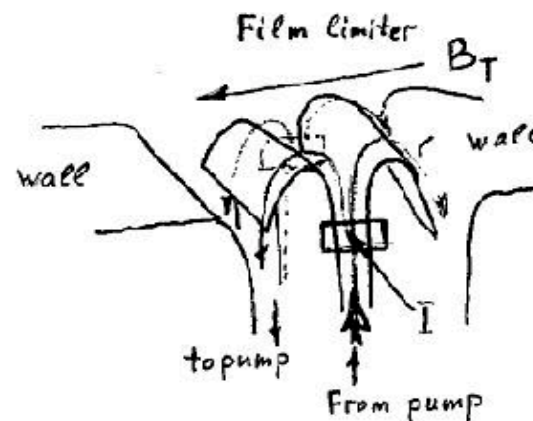
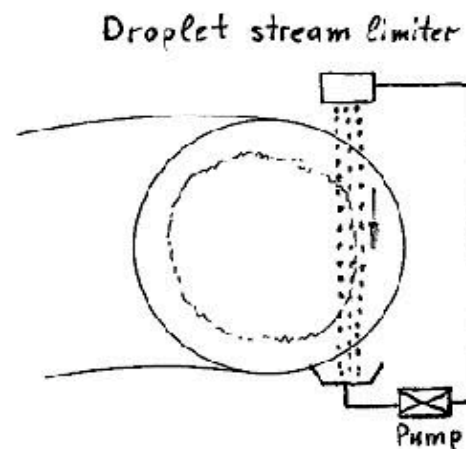
Moving-surface plasma-facing component system

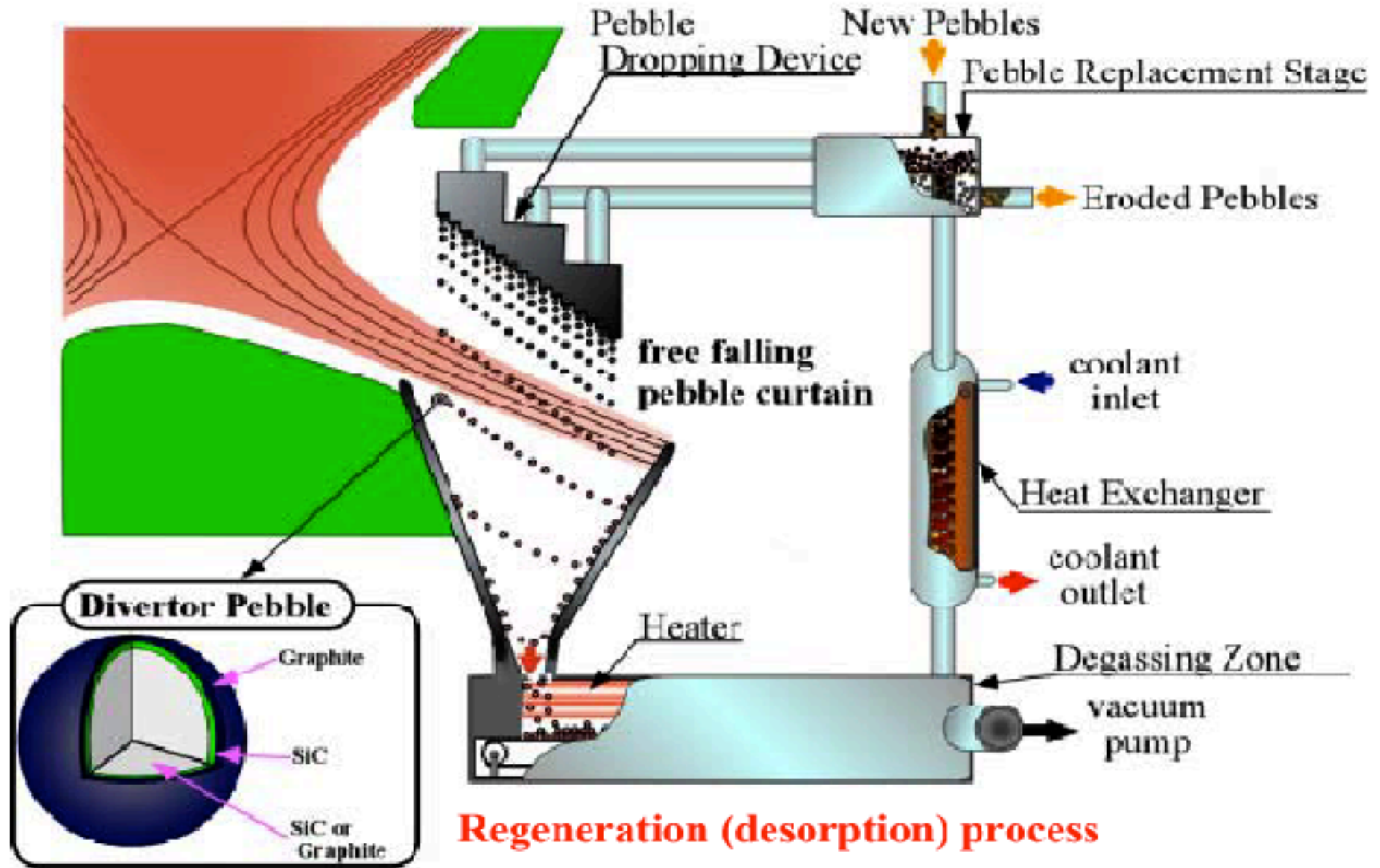
After Y. Hirooka et al., 17th SOFE, in San Diego 1997

## liquid metal wall/curtain

(Badger et al. 1974, T-3M tokamak, Troitsk, 1984-1988)

Tokamak T-3M (Shatura 1985)

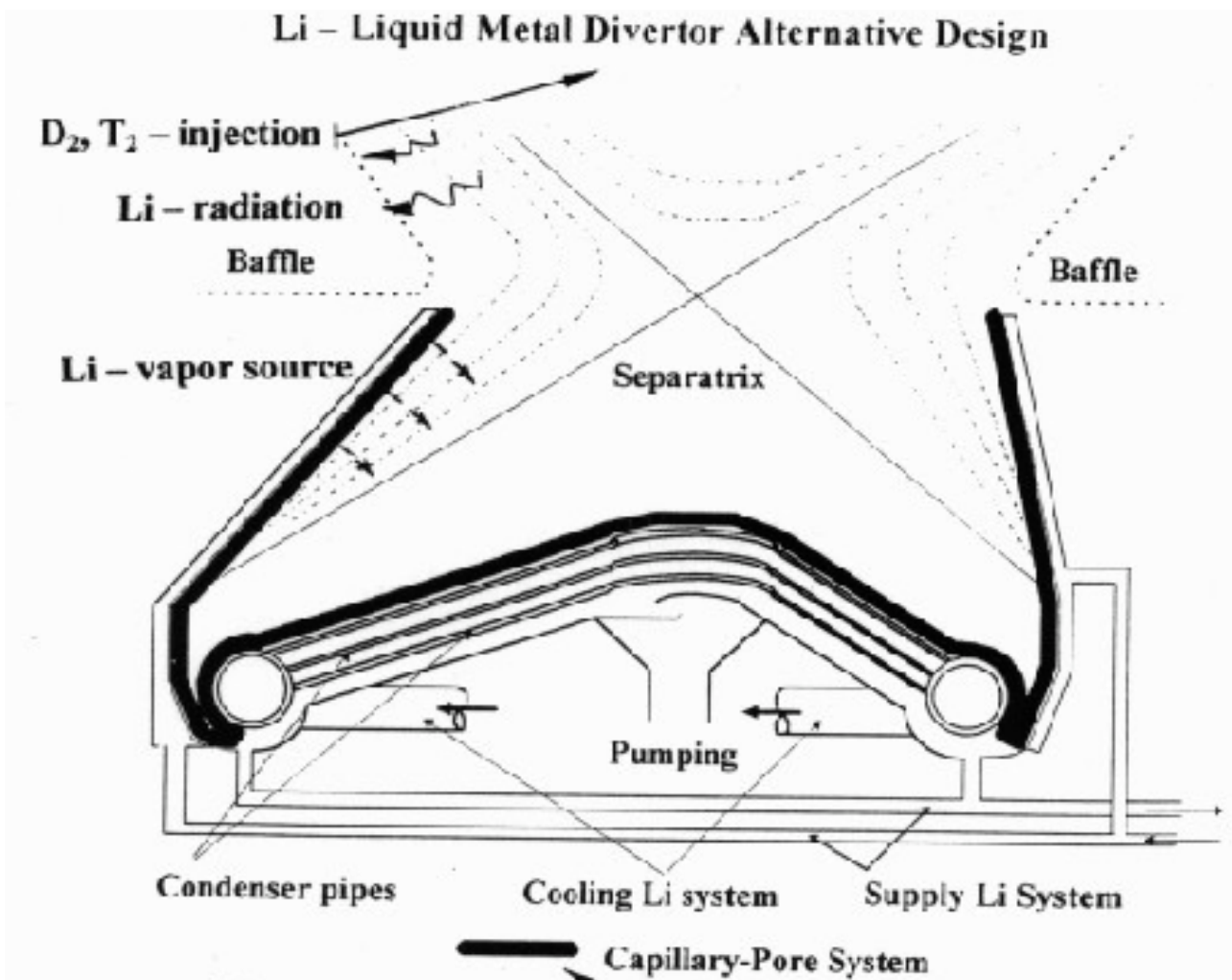




After M. Nishikawa, J. Plasma and Fusion Res. 78(2002)129.



porous capillary divertor plates (Schorn et al. 1989, Mirnov et al. 2001)



scheme of “Radiative Li Divertor” with porous capillary divertor plates for ITER-like tokamaks

Gallium droplet limiter (experiments were done on T-3M tokamak in Shatura, in 1984-85.  
 Gallium film (gravity driven) limiter (experiments were done on T-3M in Shatura, in 1986-88)

development of new material systems, for example:

Lithium is continuously pressed through a CFC material with a rather loose composite of fibers (i.e., with reduced matrix material)

Large thermal loads caused by **disruptions or ELMs** have to be absorbed by the Li vapor cloud formed immediately after impact of the plasma particles from the disrupted plasma.

The CFC material should efficiently conduct the heat along the fibers toward the heat sink of the cooling system installed beneath the targets. The **chemical erosion of graphite** would be significantly reduced.

Lithium would assist in effective **wall pumping (means low recycling !!!)**.

Both materials, as low-Z elements, **radiate** almost completely in the plasma edge.

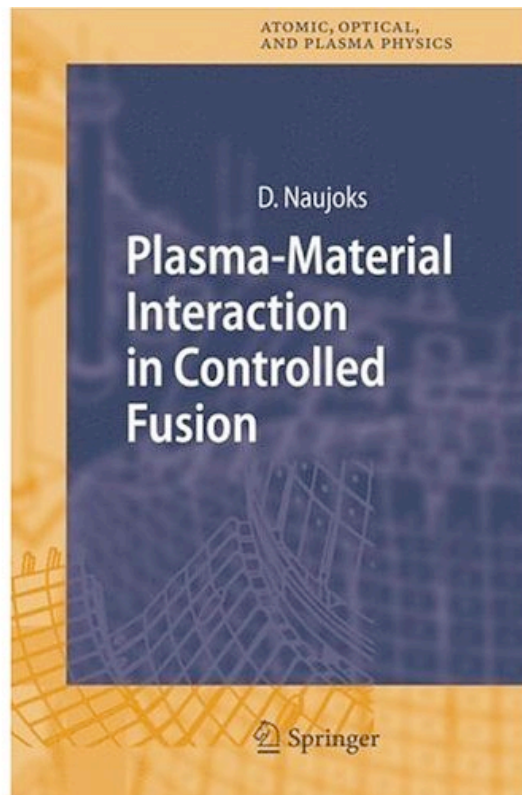
it may be possible to use the outstanding properties of both materials, while compensating for their handicaps

There are still major gaps in our present understanding.  
Among them are:

- (1) the lack of material data and rate coefficients,
- (2) the carbon–tritium cycle, T retention due to damage induced by neutrons, gaps
- (3) the tritium/carbon layer removal (by oxygen, lasers, scavenger techniques),
- (4) the dust problem (radioactive content (T, W), reactivity, explosion hazard),
- (5) the behavior of mixed materials (chemical composition, change of properties),
- (6) contribution of arcing to the erosion processes,
- (7) melting of tile edges (hot spot mitigation), crack formation (macrobrush)
- (8) ELM mitigation, disruption suppression,
- (9) the effects of neutron irradiation.

In general, a reliable solution of the material problem can only be found in the joint development of new materials and suitable plasma scenarios.

Despite all experience in plasma control that will be gathered during the next few years, one concern remains — the high flux of neutrons.



**Plasma-Material Interaction in Controlled Fusion**

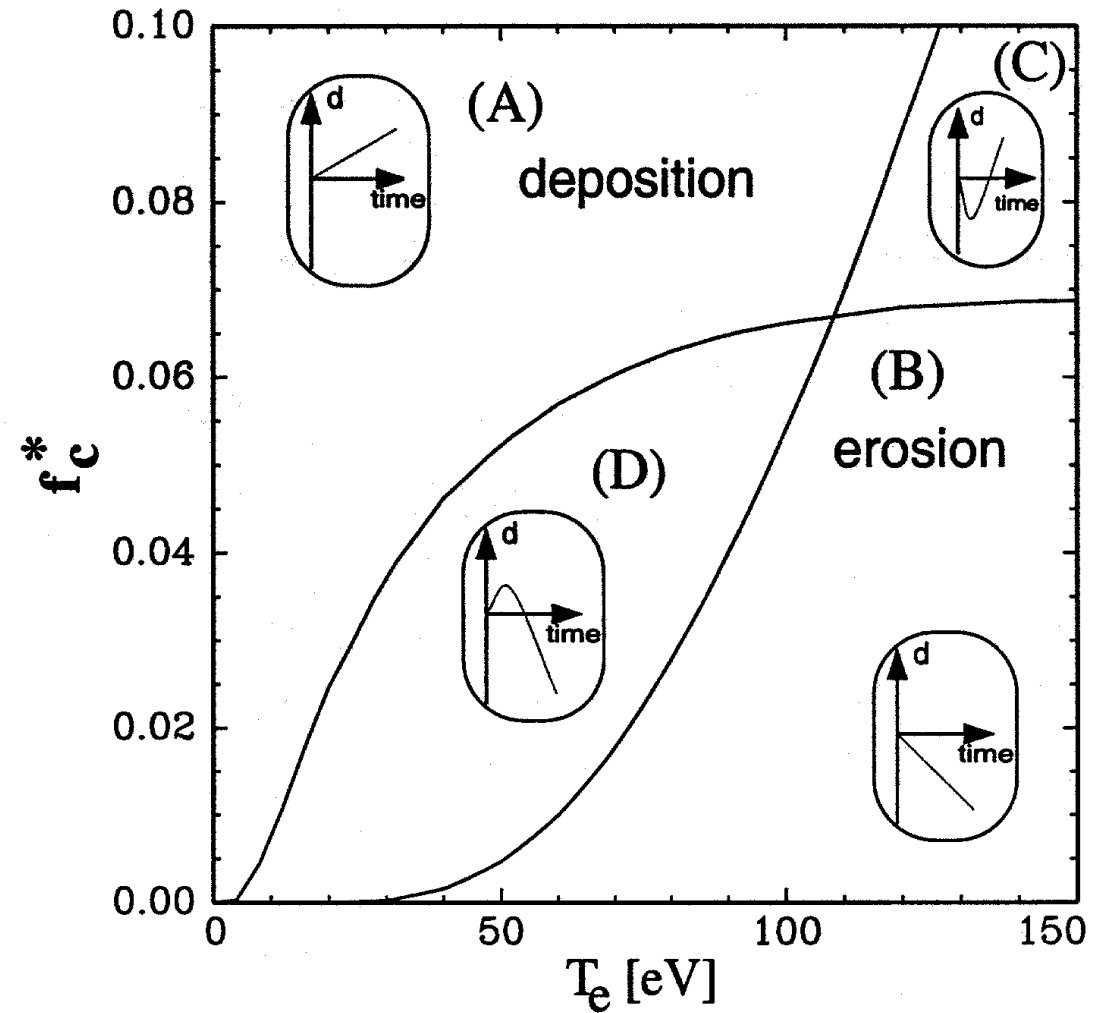
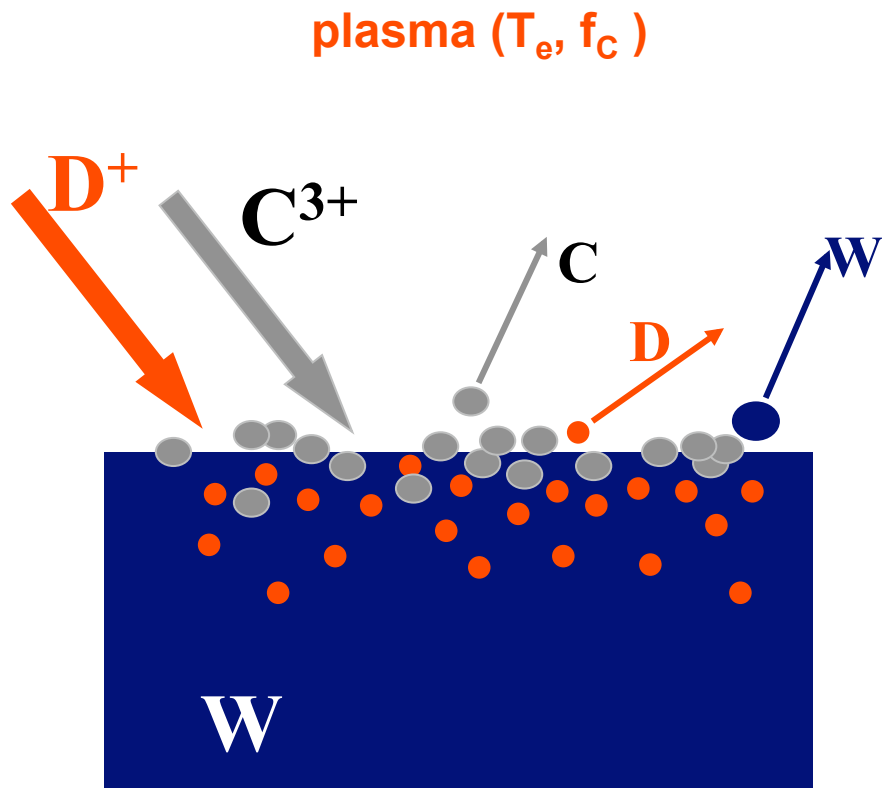
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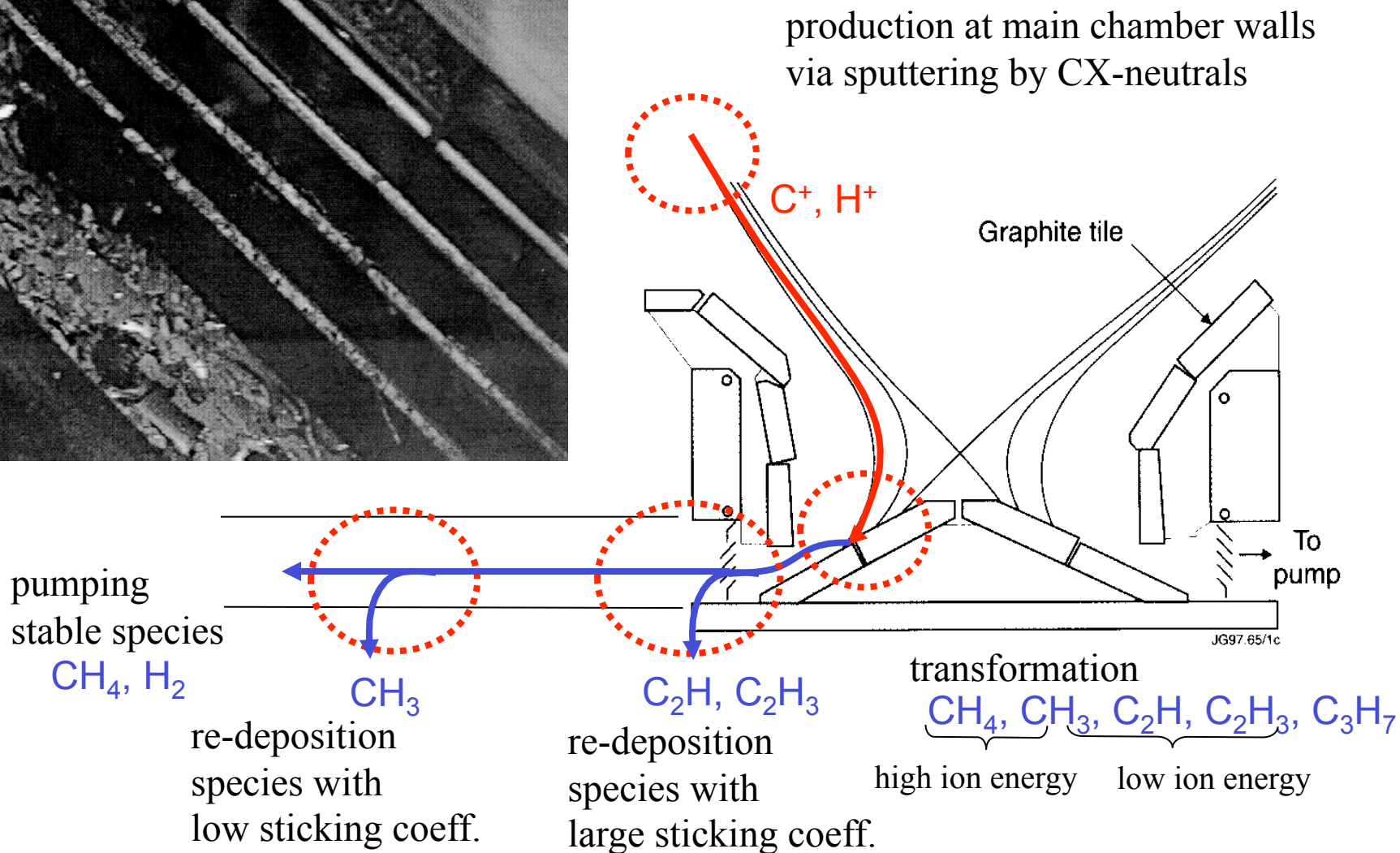
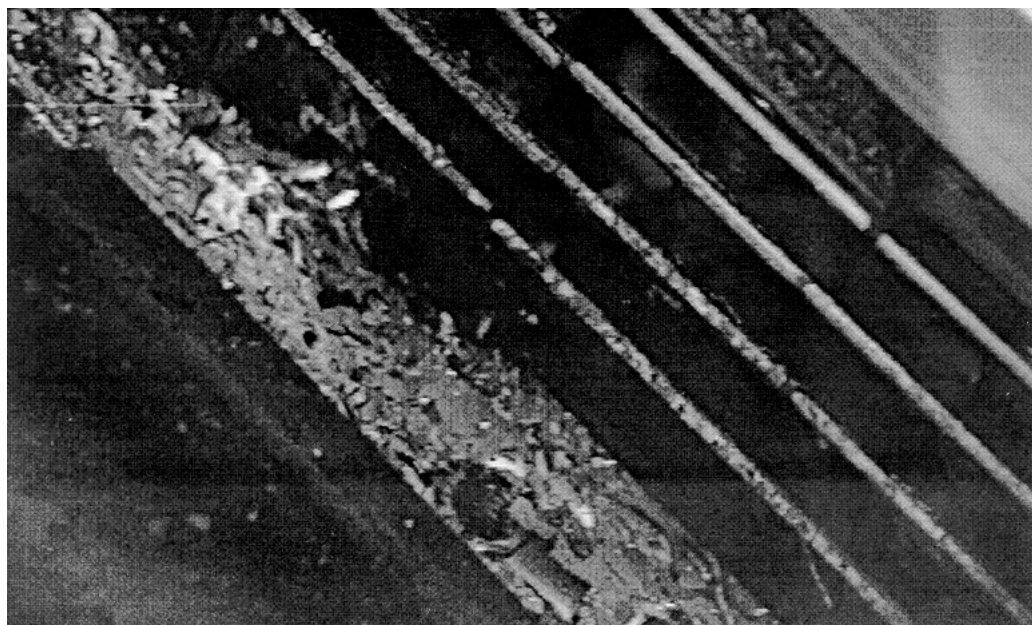
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**About this book**

Plasma-Material Interaction in Controlled Fusion deals with the specific contact between the fourth state of matter, i.e. plasma, and the first state of matter, i.e. a solid wall, in controlled fusion experiments. A comprehensive analysis of the main processes of plasma-surface interaction is given together with an assessment of the most critical questions within the context of general criteria and operation limits. It is shown that the choice of plasma-facing materials is reduced to a very limited list of possible candidates. Plasma-Material Interaction in Controlled Fusion emphasizes that a reliable solution of the material problem can only be found by adjusting the materials to suitable plasma scenarios and vice versa.

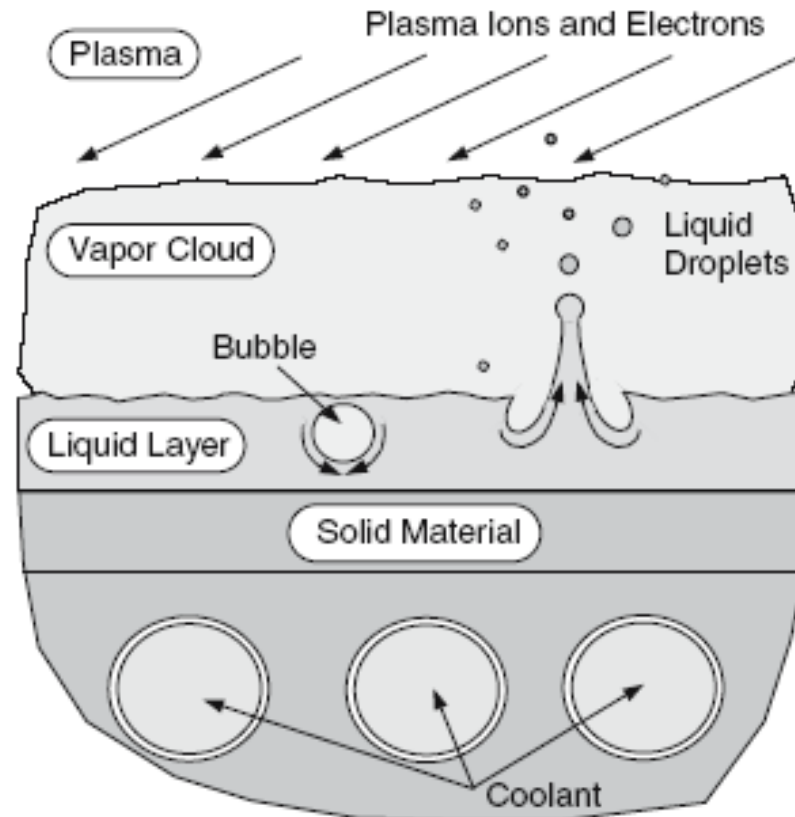


advanced erosion model for multi-component targets



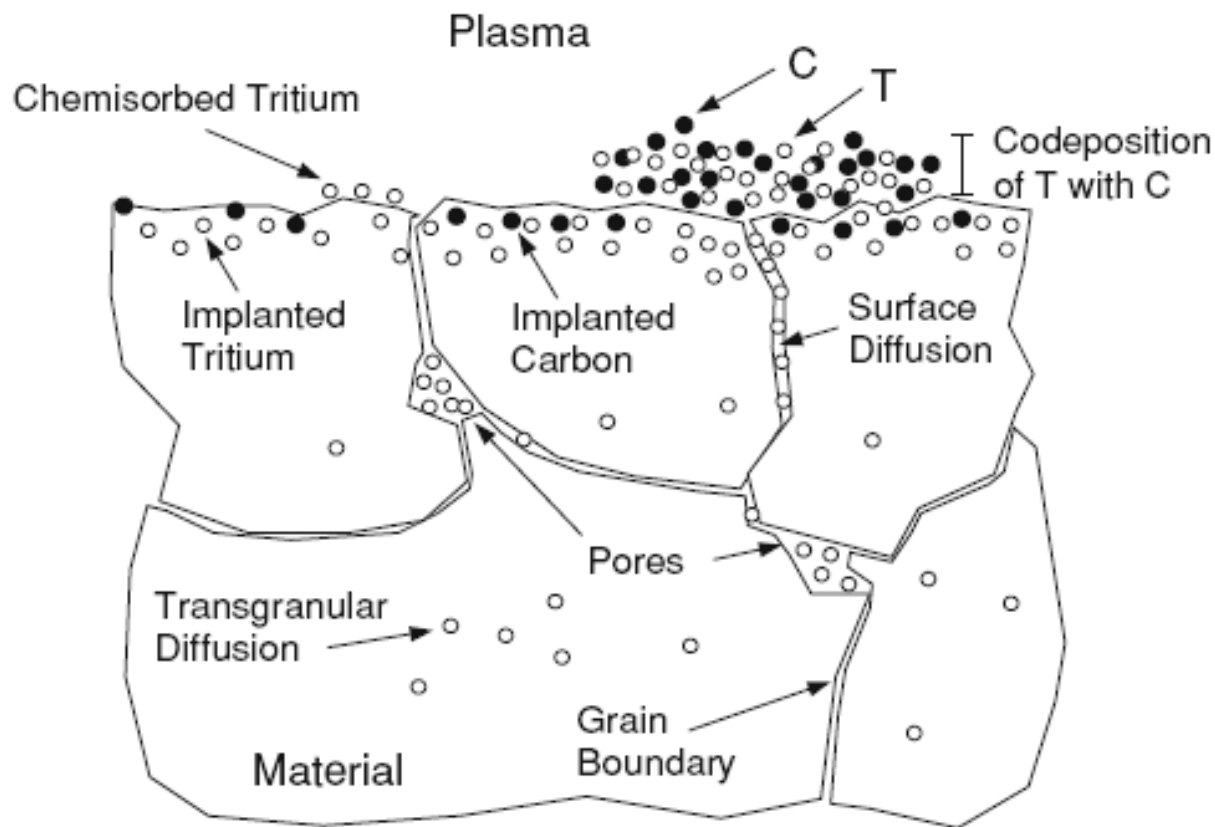
thick deposits ('flakes') found in remote areas of JET

in addition to the elementary erosion processes, phenomena observed in present fusion experiments such as plasma disruptions, ELMs, hot spots, dust production, and erosion due to runaway electrons, alpha particle, and charge-exchange neutrals can generate large influxes of impurities into the plasma



different interaction zones during large heat loads onto the material, for example, during plasma disruptions or ELMs

The main processes are **defect production** and **agglomeration**, **compositional changes of alloys by preferential sputtering**, **Gibbsian segregation**, **displacement mixing**, **bombardment-induced decomposition**, **radiation-enhanced diffusion**, and **phase transformation**.



schematic of different retention and diffusion channels of tritium



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The End

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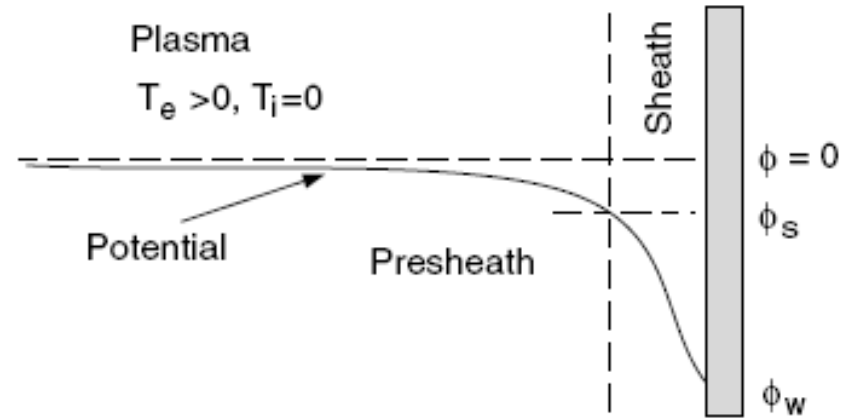
ion flux density

$$\Gamma_{i,s} = n_{pl,s} v_{i,s} \quad v_{i,s} = \sqrt{\frac{-2e\phi_s}{m_i}}$$

electron flux density

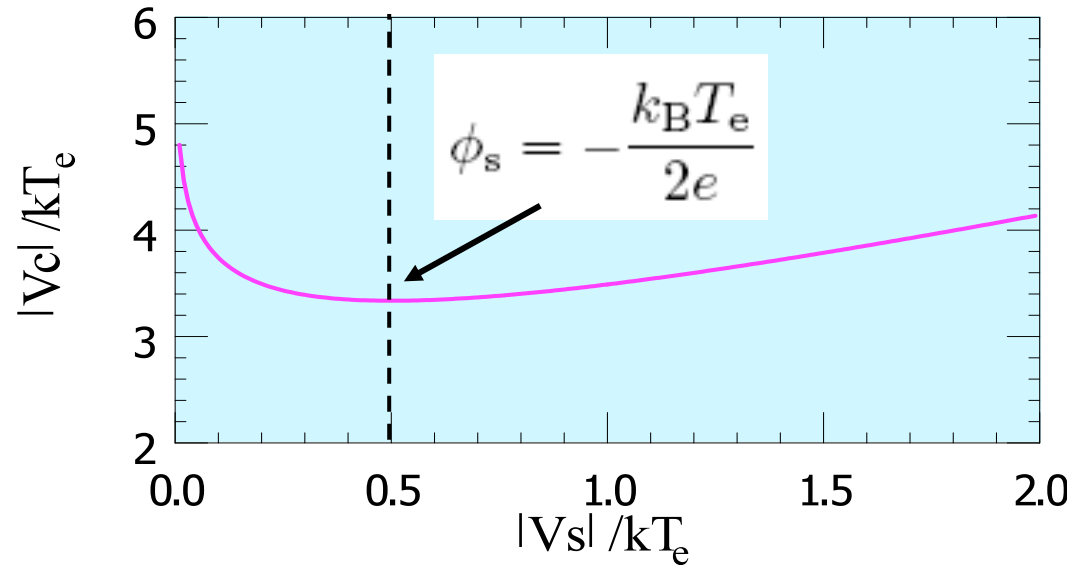
$$\Gamma_{e,w} = \frac{n_{pl,s}}{4} \langle v_e \rangle \exp\left\{\frac{e(\phi_w - \phi_s)}{k_B T_e}\right\}$$

ambipolarity condition  $\Gamma_{i,w} = \Gamma_{e,w}$

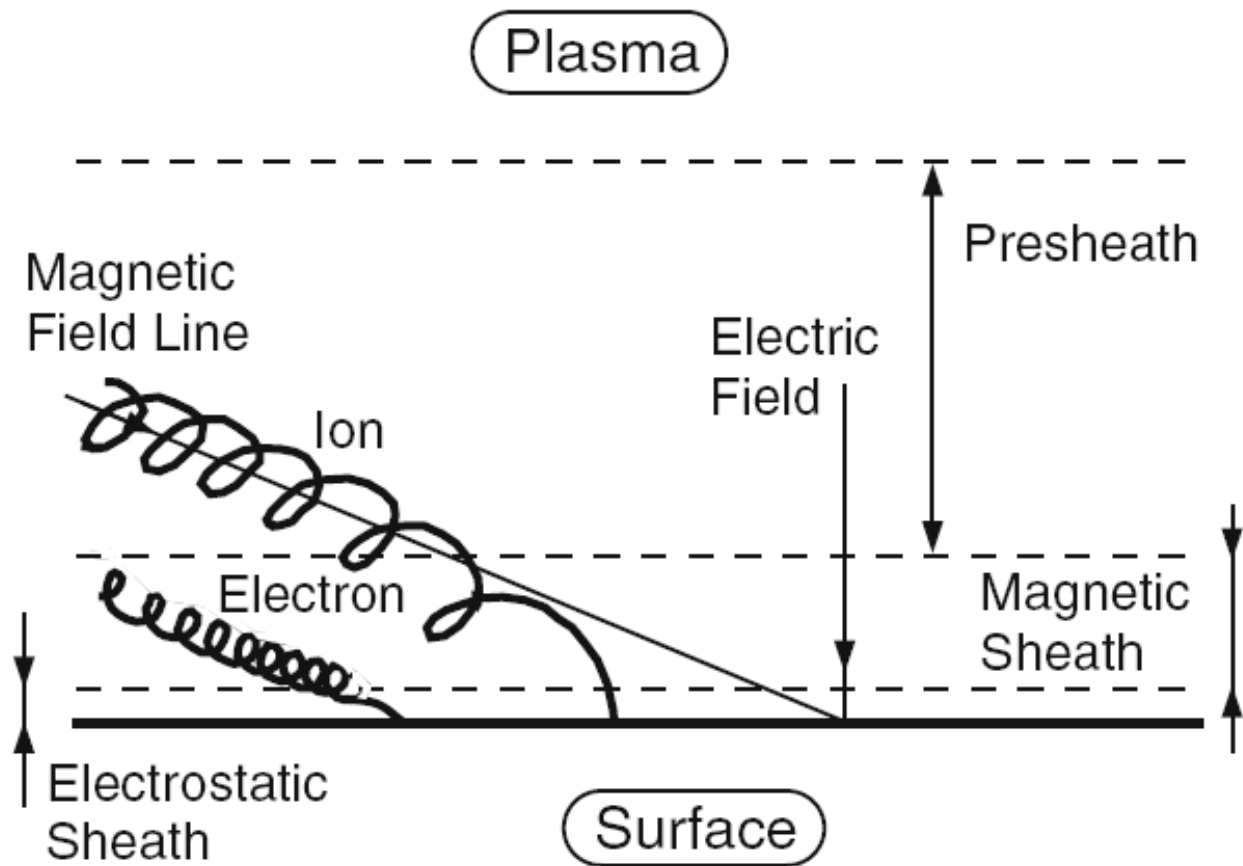


$$\begin{aligned} \phi_w &= \phi_s + \frac{k_B T_e}{e} \ln\left(\frac{4v_{i,s}}{\langle v_e \rangle}\right) \\ &= \phi_s + \frac{k_B T_e}{2e} \left[ \ln\left(\frac{4\pi m_e}{m_i}\right) + \ln\left(\frac{-e\phi_s}{k_B T_e}\right) \right] \\ \phi_w &= \frac{k_B T_e}{2e} \left[ \ln\left(\frac{2\pi m_e}{m_i}\right) - 1 \right] \quad \text{wall potential} \end{aligned}$$

$$v_{i,s} = \sqrt{\frac{k_B T_e}{m_i}} = c_s$$



Bohm criterion with the = sign ! according to a energy minimum argument



schematic of the different plasma zones in the near-surface region